SYLLABUS

Section 1: Mathematical Physics

Vector calculus: linear vector space: basis, orthogonality and completeness; matrices; similarity transformations, diagonalization, eigen values and eigen vectors; linear differential equations: second order linear differential equations and solutions involving special functions; complex analysis: Cauchy-Riemann conditions, Cauchy's theorem, singularities, residue theorem and applications; Laplace transform, Fourier analysis; elementary ideas about tensors: covariant and contravariant tensors.

Section 2: Classical Mechanics

Lagrangian formulation: D'Alembert's principle, Euler-Lagrange equation, Hamilton's principle, calculus of variations; symmetry and conservation laws; central force motion: Kepler problem and Rutherford scattering; small oscillations: coupled oscillations and normal modes; rigid body dynamics: interia tensor, orthogonal transformations, Euler angles, Torque free motion of a symmetric top; Hamiltonian and Hamilton's equations of motion; Liouville's theorem; canonical transformations: action-angle variables, Poisson brackets, Hamilton-Jacobi equation.

Special theory of relativity: Lorentz transformations, relativistic kinematics, mass-energy equivalence.

Section 3: Electromagnetic Theory

Solutions of electrostatic and magnetostatic problems including boundary value problems; method of images; separation of variables; dielectrics and conductors; magnetic materials; multipole expansion; Maxwell's equations; scalar and vector potentials; Coulomb and Lorentz gauges; electromagnetic waves in free space, non-conducting and conducting media; reflection and transmission at normal and oblique incidences; polarization of electromagnetic waves; Poynting vector, Poynting theorem, energy and momentum of electromagnetic waves; radiation from a moving charge.

Section 4: Quantum Mechanics

Postulates of quantum mechanics; uncertainty principle; Schrodinger equation; Dirac Bra-Ket notation, linear vectors and operators in Hilbert space; one dimensional potentials: step potential, finite rectangular well, tunneling from a potential barrier, particle in a box, harmonic oscillator; two and three dimensional systems: concept of degeneracy; hydrogen atom; angular momentum and spin; addition of angular momenta; variational method and WKB approximation, time independent perturbation theory; elementary scattering theory, Born approximation; symmetries in quantum mechanical systems.

Section 5: Thermodynamics and Statistical Physics

Laws of thermodynamics; macrostates and microstates; phase space; ensembles; partition function, free energy, calculation of thermodynamic quantities; classical and quantum statistics; degenerate Fermi gas; black body radiation and Planck's distribution law; BoseEinstein condensation; first and second order phase transitions, phase equilibria, critical point.

Section 6: Atomic and Molecular Physics

Spectra of one-and many-electron atoms; spin-orbit interaction: LS and jj couplings; fine and hyperfine structures; Zeeman and Stark effects; electric dipole transitions and selection rules; rotational and vibrational spectra of diatomic molecules; electronic transitions in diatomic molecules, Franck-Condon principle; Raman effect; EPR, NMR, ESR, X-ray spectra; lasers: Einstein coefficients, population inversion, two and three level systems.

Section 7: Solid State Physics

Elements of crystallography; diffraction methods for structure determination; bonding in solids; lattice vibrations and thermal properties of solids; free electron theory; band theory of solids: nearly free electron and tight binding models; metals, semiconductors and insulators; conductivity, mobility and effective mass; Optical properties of solids; Kramer's-Kronig relation, intra- and inter-band transitions; dielectric properties of solid; dielectric function, polarizability, ferroelectricity; magnetic properties of solids; dia, para, ferro, antiferro and ferri-magnetism, domains and magnetic anisotropy; superconductivity: Type-I and Type II superconductors, Meissner effect, London equation, BCS Theory, flux quantization.

Section 8: Electronics

Semiconductors in equilibrium: electron and hole statistics in intrinsic and extrinsic semiconductors; metal-semiconductor junctions; Ohmic and rectifying contacts; PN diodes, bipolar junction transistors, field effect transistors; negative and positive feedback circuits; oscillators, operational amplifiers, active filters; basics of digital logic circuits, combinational and sequential circuits, flip-flops, timers, counters, registers, A/D and D/A conversion.

Section 9: Nuclear and Particle Physics

Nuclear radii and charge distributions, nuclear binding energy, electric and magnetic moments; semi-empirical mass formula; nuclear models; liquid drop model, nuclear shell model; nuclear force and two nucleon problem; alpha decay, beta-decay, electromagnetic transitions in nuclei; Rutherford scattering, nuclear reactions, conservation laws; fission and fusion; particle accelerators and detectors; elementary particles; photons, baryons, mesons and leptons; quark model; conservation laws, isospin symmetry, charge conjugation, parity and time-reversal invariance.

EXAM PATTERN

Sections	General Aptitude	Subject (PH)
Number of Questions	**10**	**55**
Marks	5 ques. of 1 mark each and 5 ques. of 2 marks each.	25 ques. of 1 mark each and 30 ques. of 2 marks each.
Negative Marking	**For 1 mark questions:** 1/3 mark will be deducted; **For 2 marks questions:** 2/3 marks will be deducted.	**For 1 mark questions:** 1/3 mark will be deducted; **For 2 marks questions:** 2/3 marks will be deducted; No negative marking for MSQ & NAT
Types of Question	MCQ	MCQ, MSQ & NAT
Mode of Exam	Online	
Total number of Question	65	
Total Marks	**100**	

GATE PHYSICS EXAM CUT-OFF ANALYSIS

GATE CUT-OFF	GEN/EWS	OBC (NCL)	SC/ST/PWD
2023	31.1	27.9	20.7
2022	26.5	23.8	17.6
2021	32.8	29.5	21.8
2020	37.2	33.4	24.8

INDEX

GATE - PHYSICS

MATHEMATICAL PHYSICS: NUMBER OF QUESTIONS ASKED IN PREVIOUS EXAMS															
SUB - UNIT		2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
1	VECTOR SPACE	2	2	2	1	2	2	-	2	-	-	-	1	1	1
2	MATRICES	2	1	1	1	-	-	1	1	1	4	1	1	1	1
3	LINEAR AND ORDINARY DIFFERENTIAL EQUATION	1	-	-	1	2	1	1	1	1	1	1	2	1	1
4	SPECIAL FUNCTIONS	2	1	1	1	2	1	2	1	2	1	1	2	2	1
5	LAPLACE TRANSFORM	-	-	-	-	1	-	-	-	-	-	-	1	-	-
6	FOURIER SERIES	-	-	1	-	-	1	1	-	1	2	-	-	-	1
7	ELEMENTARY IDEAS OF TENSORS	-	1	1	-	-	-	-	1	-	1	-	-	1	-
8	DIRAC DELTA	-	-	-	1	1	-	-	-	-	-	-	-	-	-

CHAPTER 1

MATHEMATICAL PHYSICS

GATE PHYSICS

(PREVIOUS YEAR EXAM QUESTIONS)

VECTOR SPACE

[GATE 2011]

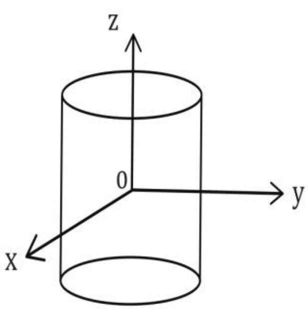
1. The unit vector normal to the surface $x^2 + y^2 - z = 1$ at the point $P(1, 1, 1)$ is

(A) $\frac{\hat{\imath}+\hat{\jmath}-\hat{k}}{\sqrt{3}}$ (B) $\frac{2\hat{\imath}+\hat{\jmath}-\hat{k}}{\sqrt{6}}$

(C) $\frac{\hat{\imath}+2\hat{\jmath}-\hat{k}}{\sqrt{6}}$ (D) $\frac{2\hat{\imath}+2\hat{\jmath}-\hat{k}}{\sqrt{6}}$

[GATE 2011]

2. Consider a cylinder of height h and radius a a closed at both ends, centered at the origin. Let $\vec{r} = \hat{\imath}x + \hat{\jmath}y + \hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\vec{r}.\hat{n}\, ds$ over the closed surface of the cylinder is



(A) $2\pi a^2(a + h)$ (B) $3\pi a^2 h$

(C) $2\pi a^2 h$ (D) Zero

[GATE 2012]

3. Identify the CORRECT statements for the following vectors $\bar{a} = 3\hat{\imath} + 2\hat{\jmath}$ and $\bar{b} = \hat{\imath} + 2\hat{\jmath}$.

(A) The vectors $\bar{a}$ and $\bar{b}$ are linearly independent.

(B) The vectors $\bar{a}$ and $\bar{b}$ are linearly dependent.

(C) The vectors $\bar{a}$ and $\bar{b}$ are orthogonal.

(D) The vectors $\bar{a}$ and $\bar{b}$ are normalized.

[GATE 2012]

4. Given $\bar{F} = \bar{r} \times \bar{B}$ where $\bar{B} = B_0(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a constant vector and $\bar{r}$ is the position vector. The value of $\oint \bar{F}.d\bar{r}$ where C is circle of unit radius centered at origin is

(A) 0 (B) $2\pi B_0$

(C) $-2\pi B_0$ (D) 1

[GATE 2013]

5. For a scalar function φ satisfying the Laplace equation, $\nabla\varphi$ has

(A) Zero curl and non-zero divergence

(B) Non-zero curl and zero divergence

(C) Zero curl and zero divergence

(D) Non-zero curl and non-zero divergence

[GATE 2013]

6. If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\nabla(\vec{A} \cdot \vec{B} \times \vec{r})$ is

(A) $\vec{A} \cdot \vec{B}$ (B) $\vec{A} \times \vec{B}$

(C) $\vec{r}$ (D) zero

[GATE 2014]

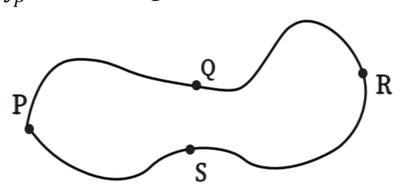
7. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1, 1, 1) is

(A) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (B) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$

(C) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$ (D) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

[GATE 2015]

8. Given that the magnetic Flux through the closed loop PQRSP is ϕ. If $\int_p^R \vec{A} \cdot \vec{dl} = \phi_1\ along$ PQR, the value of $\int_P^R \vec{A} \cdot \vec{dl}$ along PSR is



(A) $\phi - \phi_1$ (B) $\phi_1 - \phi$

(C) $-\phi_1$ (D) ϕ_1

[GATE 2015]

9. Four face are given below in Cartesian and spherical polar coordinates.

(i) $\vec{F}_1 = K\, exp(-r^2/R^2)\hat{r}$

(ii) $\vec{F}_2 = K(x^3\hat{y} - y^3\hat{z})$

(iii) $\vec{F}_3 = K(x^3\hat{x} - y^3\hat{y})$

(iv) $\vec{F}_4 = K(\hat{\phi}/r)$

Where K is a constant. Identify the correct option.

(A) (iii) and (iv) are conservative but (1) and (ii) are not

(B) (i) and (ii) are conservative but (iii) and (iv) are not

(C) (ii) and (iii) are conservative but (i) and (iv) are not

(D) (i) and (iii) are conservative but (ii) and (iv) are not

[GATE 2016]

10. Let V_i be the t^{th} component of a vector field $\vec{V}$, which has zero divergence. $If\ \partial_j, = \partial/\partial x_j)$, the expression for $\in_{ijk}\in_{ijk}\ \partial_j \partial_l V_m$ is equal to

(A) $-\partial_j\partial_k\partial_i$ (B) $\partial_j\partial_k\partial_i$
(C) $\partial_j^2 V_i$ (D) $-\partial_j^2 V_i$

[GATE 2016]

11. The direction of $\vec{\nabla}f$ for a scalar field $(x,y,z) = \frac{1}{2}x^2 - xy + \frac{1}{2}z^2$ at the point $P(1,1,2)$ is

(A) $\frac{(-\hat{j}-2\hat{k})}{\sqrt{5}}$ (B) $\frac{(-\hat{j}+2\hat{k})}{\sqrt{5}}$
(C) $\frac{(\hat{j}-2\hat{k})}{\sqrt{5}}$ (D) $\frac{(\hat{j}+2\hat{k})}{\sqrt{5}}$

[GATE 2018]

12. In spherical polar coordinates(r,θ,ϕ), the unit vector $\hat{\theta}$ at $\left(10,\frac{\pi}{4},\frac{\pi}{2}\right)$ is

(A) $\hat{k}$ (B) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$
(C) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (D) $\frac{1}{\sqrt{2}}(\hat{j}-\hat{k})$

[GATE 2018]

13. Given $\vec{V}_1 = \hat{\imath} - \hat{j}$ and $\vec{V}_2 = -2\hat{\imath} + 3\hat{j} + 2\hat{k}$, which one of the following $\vec{V}_3$ makes $(\vec{V}_1, \vec{V}_2, \vec{V}_3)$ a complete set for a three dimensional real linear vector space?

(A) $\vec{V}_3 = \hat{\imath} + \hat{j} + 4\hat{k}$
(B) $\vec{V}_3 = 2\hat{\imath} - \hat{j} + 2\hat{k}$
(C) $\vec{V}_3 = \hat{\imath} + 2\hat{j} + 6\hat{k}$
(D) $\vec{V}_3 = 2\hat{\imath} + \hat{j} + 4\hat{k}$

[GATE 2022]

14. From the pairs of operators given below, identify the ones which commute. Here l and j correspond to the orbital angular momentum and the total angular momentum, respectively.

(A) l^2, j^2 (B) l^2, j_z
(C) j^2, l_z (D) l_z, j_z

[GATE 2023]

15. Consider the vector field $\vec{V}$ consisting of the velocities of points on a thin horizontal disc of radius $R = 2\ m$, moving anticlockwise with uniform angular speed $\omega = 2\ rad/sec$ about an axis passing through its center. If $V = |\vec{V}|$, then which of the following options is (are) CORRECT? (In the options, $\hat{r}$ and $\hat{\theta}$ are unit vectors corresponding to the plane polar coordinates r and θ).

You may use the fact that in cylindrical coordinates (s,ϕ,z) (s is the distance from the z-axis), the gradient, divergence, curl and Laplacian operators are:

$\vec{\nabla}f = \frac{\partial f}{\partial s}\hat{S} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{Z};$

$\vec{\nabla}.\vec{A} = \frac{1}{s}\frac{\partial}{\partial s}(sA_s) + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_Z}{\partial Z};$

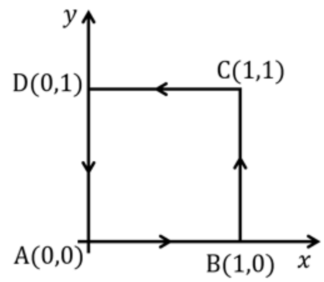
$\vec{\nabla}\times\vec{A} = \left(\frac{1}{s}\frac{\partial A_Z}{\partial \phi} - \frac{\partial A_\phi}{\partial Z}\right)\hat{S} + \left(\frac{\partial A_S}{\partial Z} - \frac{\partial A_Z}{\partial S}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial}{\partial s}(sA_\phi) - \frac{\partial A_S}{\partial \phi}\right)\hat{Z};$

$\vec{\nabla}^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2};$

(A) $\vec{\nabla}V = 2\hat{r}$
(B) $\vec{\nabla}.\vec{V} = 2$
(C) $\vec{\nabla}\times\vec{V} = 4\hat{Z}$, where $\hat{Z}$ is a unit vector perpendicular to the (r,θ) plane
(D) $\vec{\nabla}^2 V = -\frac{4}{3}$ at $r = 1.5\ m$

[GATE-2024]

16. Consider a vector field F = (2xz + 3y²)ŷ+4yz$^2\hat{z}$. The closed path (T: A → B → C→D → A) in z = 0 plane is shown in figure.



$\phi_r\vec{F}.\vec{dl}$ denotes the line integral of $\vec{F}$ along the closed path r. Which of the following option is/are true?

(A) $\phi_r\vec{F}.\vec{dl}$=0
(B) $\vec{F}$ is non –conservative.
(C) $\vec{\nabla}.\vec{F}$=0
(D) $\vec{F}$can be written as the gradient of a scalar field

ANSWER KEY									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
D	B	A	C	C	B	D	B	D	D
11	**12**	**13**	**14**	**15**	**16**				
D	D	D	A,B,D	A,C,D	A,B				

SOLUTIONS: VECTOR SPACE

1. **Solution: (D)**

We know that,
Unit vector normal to the surface

$\hat{n} = \frac{\vec{\nabla}\phi}{|\vec{\nabla}\phi|}\bigg|_{(11,1)} = \frac{2\hat{\imath}+2\hat{j}-\hat{k}}{3}$

2. **Solution: (B)**

By using divergence theorem we get

$\int_S \vec{r}\cdot\hat{n}dS = \int_V (\vec{\nabla}\cdot\vec{r})dV$
$= 3\int_V dV = 3V = 3\pi a^2 h$

3. **Solution: (A)**

Given vectors are $\vec{a} = 3\hat{\imath} + 2\hat{j},\ \vec{b} = \hat{\imath} + 2\hat{j}$

$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} \neq 0$

Therefore $\vec{a}$ and $\vec{b}$ are linearly independent
Further, $\vec{a}.\vec{b} \neq 0$, $\vec{a}$ and $\vec{b}$ are not orthogonal
$|\vec{a}| \neq, |\vec{b}| \neq 1$ Not normalized

4. Solution: (C)

From given function $\vec{F} = \vec{r} \times \vec{B}$

$$\oint_C \vec{F}.\overrightarrow{dr} = \iint_S (\vec{\nabla} \times \vec{F}) = \iint [\vec{\nabla} \times (\vec{r} \times \vec{B})].\overrightarrow{dS}$$

$$= \iint_S [(\vec{B}.\vec{\nabla})\vec{r} - (\vec{r}.\vec{\nabla})\vec{B} + \vec{r}(\vec{\nabla}.\vec{B}) - B(\vec{\nabla}.\vec{r})].\overrightarrow{dS}$$

$= \iint(\vec{B} - 0 + 0 - 3\vec{B}).d\vec{S} = \iint -2\vec{B}.d\vec{S} -= -2B_0\pi =$
$-2\pi B_0$

5. Solution: (C)

Consider a scalar function $\emptyset$ which is satisfying the Laplace equation, $\overrightarrow{\nabla^2}\emptyset = 0$

$$\vec{\nabla}(\vec{\nabla}\emptyset) = 0$$

Also

$$\vec{\nabla} \times (\vec{\nabla}\emptyset) = 0$$

Hence, $\vec{\nabla}\emptyset$ has both zero divergence and zero curl.

6. Solution: (B)

$\vec{\nabla}[\vec{A}.(\vec{B} \times \vec{r})] = \vec{\nabla}[\vec{r}.(\vec{A} \times \vec{B})] = \vec{\nabla}[(\vec{A} \times \vec{B}).\vec{r}]$

If $\vec{A}$ and $\vec{B}$ are constant vectors, then$(\vec{A} \times \vec{B})$ is also constant vector.

For constant vector $\vec{a}, \vec{\nabla}(\vec{a}.\vec{r}) = \vec{a}$

Therefore, $\vec{\nabla}[(\vec{A} \times \vec{B}).\vec{r}] = (\vec{A} \times \vec{B})$

7. Solution: (D)

We know that the unit vector perpendicular to the surface is given by,

$\hat{n} = \frac{\vec{\nabla}\emptyset}{|\vec{\nabla}\emptyset|}\Big|$

$= \frac{2x\hat{\imath}+2y\hat{\jmath}+2z\hat{k}}{2\sqrt{x^2y^2z^2}}\Big|_{(1,1,1)} = \frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$

8. Solution: (B)

Magnetic flux through the loop

$i.e. \iint_S \vec{B}.d\vec{S} = \emptyset \Rightarrow \iint_S (\vec{\nabla} \times \vec{A})\, dS = \emptyset, \int_{pqrs} \vec{A}.\overrightarrow{dl} = \varphi$

$\Rightarrow \varphi_1 + \int_{PQR} \vec{A}.\overrightarrow{dl} + \int_{RSP} \vec{A}.\overrightarrow{dl} = \varphi$

$\Rightarrow \varphi_1 + \int_{RSP} \vec{A}.\overrightarrow{dl} = \varphi - \int_{RSP} \vec{A}.\overrightarrow{dl} = (\varphi -$
$\varphi_1) \int_{PSR} \vec{A}.\overrightarrow{dl} = (\varphi_1 - \varphi)$

9. Solution: (D)

By taking curl to the given functions we get,

$$\vec{\nabla} \times \overrightarrow{F_1} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & \hat{r}\theta & r\sin\theta\hat{\emptyset} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \emptyset} \\ k.\exp\left(-\frac{r^2}{R^2}\right) & 0 & 0 \end{vmatrix} = 0$$

$$\vec{\nabla} \times \overrightarrow{F_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & kx^3 & -kz^3 \end{vmatrix} = 3kx^2\hat{k}$$

$$\vec{\nabla} \times \overrightarrow{F_3} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ kx^3 & ky^3 & 0 \end{vmatrix} = 0$$

$$\vec{\nabla} \times \overrightarrow{F_4} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\emptyset} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \emptyset} \\ 0 & 0 & \frac{k}{r} \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left[-\frac{k}{r^2}\right] = -\frac{k}{r^2\sin\theta}\,\hat{\theta}$$

10. Solution: (D)

As given,

$\epsilon_{ijk}\, \epsilon_{tmk}\partial_j\, \partial_t\, V_m$

$= \epsilon_{kij}\, \epsilon_{klm}\, \partial_j\, \partial_t V_m$

$= (\delta_{if}\, \delta_{jm} - \delta_{im}\, \delta_{it})\partial_j\, \partial_t\, V_m$

$= \partial_j\, \partial_i\, V_m - \partial_j\, \partial_j\, V_i = \partial_i(\partial_j\, V_j) - \partial_j^2 V_i = -\partial_j^2 V_i$

$(given: \partial_j V_j = 0)$

11. Solution: (D)

We have, $f(x, zy, z) = \frac{1}{2}x^2 - xy + \frac{1}{2}z^2$

$\therefore \vec{\nabla}f = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right)\left(\frac{1}{2}x^2 - xy + \frac{1}{2}z^2\right) = (x -$
$y)\hat{\imath} + (-x)\hat{\jmath} + z\hat{k}$

$\therefore \vec{\nabla}f|_{(1,1,2)} = (-\hat{\jmath} + 2\hat{k})$

Therefore, direction of $\vec{\nabla}f$ is $\frac{-\hat{\jmath}+2\hat{k}}{\sqrt{5}}$

12. Solution: (D)

In spherical polar coordinate, $\hat{\theta}$ is given by

$\hat{\theta} = \cos\theta\, 0 \cos\phi\, \hat{\imath} + \cos\theta \sin\phi\, \hat{\jmath} - \sin\theta\, \hat{k}$

Put, $r = 10, \theta = \frac{\pi}{4}, \phi = \frac{\pi}{2}$

$\Rightarrow \hat{\theta} = \cos\frac{\pi}{4}\cos\frac{\pi}{2}\hat{\imath} + \cos\frac{\pi}{4}\sin\frac{\pi}{2}\hat{\jmath} - \sin\frac{\pi}{4}\hat{k}$

$\Rightarrow \hat{\theta} = \frac{1}{\sqrt{2}}\hat{\jmath} - \frac{1}{\sqrt{2}}\,\hat{k}$

13. Solution: (D)

Given that,

$\overrightarrow{V_1} = \hat{\imath} - \hat{\jmath}, \overrightarrow{V_2} = -2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

If vectors,$\overrightarrow{V_1}, \overrightarrow{V_2}$ and $\overrightarrow{V_3}$ makes complete set of 3-D real linear vector space then $\overrightarrow{V_1}, \overrightarrow{V_2}$ and $\overrightarrow{V_3}$ should be linearly independent i.e. $\overrightarrow{V_1}, (\overrightarrow{V_2} \times \overrightarrow{V_3}) \neq 0$

Only option (d) satisfies these condition.

14. Solution: (A), (B), (D)

Here, $j^2 = \vec{j}\,.\,\vec{j} = (\vec{l} + \vec{s})\,.\,(\vec{l} + \vec{s}) = l^2\,(s^2 + 2\vec{l}\,.S$

Therefore, the commutator of option (a) s.

$[l^2, j^2] \quad = [l^2, l^2 + s^2 + 2\vec{l} \cdot \vec{s}]$

$\qquad\qquad = 0$ [Since, $[l^2, l_l] = 0$

Therefore, they commute with each other.

The commutator of option (b) is,

$[i^2, j_z] = [(l^2 + s^2 + 2\vec{l}\cdot\vec{s}), (l_z + s_z)]$ CAPEep find

$= 2[l_x s_x + l_y s_y + l_z s_z, (l_x + s_z)]$

$= 2([l_x s_x, l_z] + [l_y s_y, l_z] + [l_x s_x, s_z] + [l_y s_y, s_z])$

$= 2\left((-ihl_y)s_x + (i\hbar l_x)s_y + l_x(-i\hbar s_y) + l_y(ihs_x)\right) = 0$

Therefore, they commute with each other.

The commutator in option (c) is,

$[\vec{j}^2, l_z] = \{l^2 + s^2 + 2\vec{l}\cdot\vec{s}, l_z] = 2[l_x s_x + l_y s_y + l_z s_z, l_z]$

$= 2([l_x, l_z]s_x + [l_y, l_z]s_y) = 2(-i\hbar l_y s_x + i\hbar l_x s_y) \neq 0$

Therefore, they do not commute with each other.

The commutator in option (d) is,

$[l_z, j_l] = [l_z, l_z + s_z] = 0$

Therefore, they commute with each other.

15. Solution: (A), (C), (D)

Since $\vec{v} = \vec{\omega}\times\vec{r} = \omega r\hat{\varphi} \Rightarrow |\vec{v}| = \omega r$

A. $\vec{\nabla}v = \frac{\partial}{\partial r}(\omega r)\hat{r} = \omega\hat{r} = 2\hat{r}$

B. $\vec{\nabla}.v = \frac{\partial}{\partial\varphi}(\omega r) = 0$

C. $\vec{\nabla}\times v = \frac{1}{r}\frac{\partial}{\partial r}(rv\varphi)\hat{z} = \frac{1}{r}\frac{\partial}{\partial r}(r\times\omega r)\hat{z} = 2\omega\hat{z}$

$\because \vec{v} = \vec{\omega}\times\vec{r} = \omega r\hat{\varphi}$

$\vec{\nabla}^2 v = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v}{\partial r}) = \frac{1}{r}\frac{\partial}{\partial r}(r\omega) = \frac{\omega}{r} = \frac{2}{1.5} = 4/3$

$\because v = \omega r$

16. Solution: (A), (B)

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & (2xz + 3y^2) & 4yz^2 \end{vmatrix}$$

$= \hat{\imath}\,[\,(4z^2 - 2x)\,] - \hat{\jmath}.0 + \hat{k}\,[2z]$

$= (4z^2 - 2x)\hat{\imath} + 2z\hat{k}$

Which does not equal to zero so, Vector F is non-conservative vector field.

Now,

For $\oint \vec{F}.\vec{dl}$, we will use Stokes' curl theorem

$$\oint \vec{F}.\vec{dl} = \iint (\vec{\nabla}\times\vec{F}).\vec{dS}$$

$\iint(\vec{\nabla}\times\vec{F}).\vec{dS} = \iint_0^1 [(4z^2 - 2x)\hat{\imath} + 2z\hat{k}].(dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k})$

$= \left[2x^2z^2 - \frac{x^3}{3} + \frac{z^4}{6}\right]_0^1$

$= \left(2 - \frac{1}{3} + \frac{1}{6}\right) - \left(2 - \frac{1}{3} + \frac{1}{6}\right) = 0$

Therefore, $\oint \vec{F}.\vec{dl}$ should be equal to zero.

MATRICES

[GATE 2011]

1. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following is Not True?

(A) Det A = Det B
(B) Trace of A = Trace of B
(C) A and B have the same eigenvectors
(D) A and B have the same eigenvalues

[GATE 2011]

2. A 3 x 3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix is

(A) 18 (B) 12
(C) 9 (D) 6

[GATE 2012]

3. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

(A) 0, 1, 1 (B) $0, -\sqrt{2}, \sqrt{2}$
(C) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (D) $\sqrt{2}, \sqrt{2}, 0$

[GATE 2013]

4. The degenerate eigenvalue of the matrix $[4\ -1\ -1\ -1\ 4\ -1\ -1\ -1\ 4\ ]$ is (your answer should be an integer) _________________

[GATE 2014]

5. The matrix $A = \frac{1}{\sqrt{3}}[\,1\ 1+i\ 1-i\ -1\,]$ is

(A) orthogonal (B) symmetric
(C) anti-symmetric (D) unitary

[GATE 2017]

6. Let X be a column vector of dimension $n > 1$ with at least one non-zero entry. The number of non-zero eigenvalues of the matrix $M = XX^T$ is

(A) 0 (B) n
(C) 1 (D) $n-1$

[GATE 2018]

7. The eigenvalues of a Hermitian matrix are all

(A) real (B) imaginary
(C) of modulus one (D) real and positive

[GATE 2019]

8. During a rotation, vectors along the axis of rotation remain unchanged. For the rotation matrix $(0\ 1\ 0\ 0\ 0\ -1\ -1\ 0\ 0\ )$, the unit vector along the axis of rotation is

(A) $\frac{1}{3}(2\hat{\imath} - \hat{\jmath} + 2\hat{k})$ (B) $\frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} - \hat{k})$
(C) $\frac{1}{\sqrt{3}}(\hat{\imath} - \hat{\jmath} - \hat{k})$ (D) $\frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$

[GATE 2020]

9. A real, invertible 3×3 matrix M has eigenvalues $\lambda_i, (i = 1,2,3)$ and the corresponding eigenvectors are $|e_i\rangle, (i = 1,2,3)$ respectively. Which one of the following is correct?