INDEX

BARC OCES – PHYSICS QUESTION BANK

CHAPTER-1 MATHEMATICAL METHODS IN PHYSICS

PRACTICE QUESTIONS

A. VECTOR SPACE

1. If $\vec{A}(t)$ is a vector of constant magnitude, which of the following is true?
(a) $\frac{d\vec{A}}{dt} = 0$ (b) $\frac{d^2\vec{A}}{dt^2} = 0$
(c) $\frac{d\vec{A}}{dt}.\vec{A} = 0$ (d) $\frac{d\vec{A}}{dt} \times \vec{A} = 0$

2. The height of a hill at a point (x, y) in metres is given by
$h(x, y) = exp[2xy - x^2 - 2y^2 - 4x + 8y + 1]$
Where x and y are in km with respect to a certain origin. What is the unit vector in the direction of steepest ascent at the origin?
(a) $\frac{1}{\sqrt{80}}(-4\hat{\imath} + 8\hat{\jmath})$ (b) $\frac{1}{\sqrt{40}}(-8\hat{\imath} + 4\hat{\jmath})$
(c) $\frac{1}{\sqrt{80}}(4\hat{\imath} - 8\hat{\jmath})$ (d) $\frac{1}{\sqrt{40}}(4\hat{\imath} - 8\hat{\jmath})$

3. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$. Which option is correct?
(a) The three vectors are orthonormal
(b) The three vectors are linearly independent
(c) The three vectors cannot form a basis in a three-dimensional real vector space
(d) $\frac{1}{\sqrt{2}}$ (1,1,0) can be written as a linear combination of $\frac{1}{\sqrt{2}}$ (0,1,1) and $\frac{1}{\sqrt{2}}$ (1,0,1)

4. If $\vec{A} = x\hat{e}_x + y\hat{e}_y + z\,\hat{e}_z$, then $\nabla^2\vec{A}$ equals
(a) 1 (b) 3
(c) 0 (d) -3

5. Given $\overrightarrow{A} = y^2\hat{e}_x + 2yx\hat{e}_y + (xye^z - \sin x\,)\hat{e}_z$, calculate the value of $\iint_s(\nabla \times \vec{A}).\hat{n}ds$ over the part of the sphere $x^2 + y^2 + z^2 = 1$ above the xoy plane.
(a) 2 (b) 1
(c) 0 (d) -1

6. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the position vector, then the value of the following integral $\iint_s\vec{r}.\hat{n}\,dS$ is
(a) V (b) 2V
(c) 0 (d) 3V

7. Which of the following vectors is orthogonal to the vector$(a\hat{\imath} + b\hat{\jmath})$, where a and b $(a \neq b)$ are constants, and $\hat{\imath}$ and $\hat{\jmath}$ are unit orthogonal vectors?
(a) $-b\hat{\imath} + a\hat{\jmath}$ (b) $-a\hat{\imath} + b\hat{\jmath}$
(c) $-a\hat{\imath} - b\hat{\jmath}$ (d) $-b\hat{\imath} - a\hat{\jmath}$

8. The unit vector normal to the surface $3x^2 + 4y = z$ at the point (1,1,7) is
(a) $\frac{(-6\hat{\imath}+4\hat{\jmath}+\hat{k})}{\sqrt{53}}$ (b) $\frac{(4\hat{\imath}+6\hat{\jmath}-\hat{k})}{\sqrt{53}}$
(c) $\frac{(6\hat{\imath}+4\hat{\jmath}-\hat{k})}{\sqrt{53}}$ (d) $\frac{(4\hat{\imath}+6\hat{\jmath}+\hat{k})}{\sqrt{53}}$

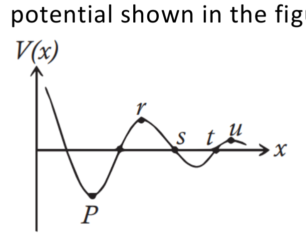
9. A vector $\vec{A} = (5x + 2y)\hat{\imath} + (3y - z)\hat{\jmath} + (2x - az)\hat{k}$ is solenoidal if the constant a has a value
(a) 4 (b) -4
(c) 8 (d) -8

10. The curl of the vector $A = zi + xj + yk$ is given by
(a) $i + j + k$ (b) $i - j + k$
(c) $i + j - k$ (d) $-i - j - k$

11. For the function $\phi = x^2y + xy$, the value of $|\vec{\nabla}\phi|$ at $x = y = 1$ is
(a) 5 (b) $\sqrt{5}$
(c) 13 (d) $\sqrt{13}$

12. The average of the function $f(x) = \sin x$ in the interval $(0, \pi)$ is
(a) $\frac{1}{2}$ (b) $\frac{2}{\pi}$
(c) $\frac{1}{\pi}$ (d) $\frac{4}{\pi}$

13. Identify the points of unstable equilibrium for the potential shown in the figure.



(a) p and s (b) q and t
(c) r and u (d) r and s

14. The average value of the function $f(x) = 4x^3$ in the interval 1 to 3 is
(a) 15 (b) 20
(c) 40 (d) 80

15. The unit normal to the curve $x^3y^2 + xy = 17$ at the point (2,0) is
(a) $\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}$ (b) $-\hat{\imath}$
(c) $-\hat{\jmath}$ (d) $\hat{\jmath}$

16. If a vector field $\vec{F} = x\hat{\imath} + 2y\hat{\jmath} + 3z\hat{k}$, then $\vec{\nabla} \times (\vec{\nabla} \times \vec{F})$ is
(a) 0 (b) $\hat{\imath}$
(c) $2\hat{\jmath}$ (d) $3\hat{k}$

17. A linear transformation T, defined as T $(x_1\ x_2\ x_3) = (x_1 + x_2\ x_2 - x_3)$, transform a vector $\vec{x}$ for a 3-dimensional real space to a 2-dimensional real space. The transformation matrix T is
(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

18. The value of $\frac{\vec{r}\cdot d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin, is
(a) 0 (b) π
(c) 4π (d) 8π

19. Consider the set of vectors in three-dimensional real vector space $R^3, S = \{(1,1,1),(1,-1,1),(1,1,-1)\}$. Which one of the following statements is true?
(a) S is not a linearly independent set
(c) The vectors in S are orthogonal
(b) S is a basis for R^3
(d) An orthogonal set of vectors cannot be generated from S

20. Let $\vec{r}$ be the position vector of a point on a closed contour C. What is the value of the line integral $\oint \vec{r}.\overrightarrow{dr}$?
(a) 0 (b) ½
(c) 1 (d) π

21. If $\bar{A}.(\bar{B} \times \bar{C}) = 0$ in 3-dimensional space, then
(a) $\bar{A}, \bar{B}, \bar{C}$ Are coplanar
(b) $\bar{A}$ is a null vector
(c) $\bar{A}, \bar{B}, \bar{C}$ span the whole 3-D space.
(d) $\bar{B} = 0$

ANSWER KEY									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
(c)	(a)	(b)	(c)	(c)	(d)	(a)	(c)	(c)	(a)
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
(d)	(b)	(c)	(c)	(d)	(a)	(a)	(c)	(a)	(a)
21									
(a)									

SOLUTION

A. VECTOR SPACE

1. Solution:
Here $\vec{A}(t).\vec{A}(t) = |\vec{A}(t)|^2$
Differentiating w.r.t. t, we get
$\Rightarrow \frac{d\vec{A}}{dt}.\vec{A} + \vec{A}.\frac{d\vec{A}}{dt} = 0 \qquad (\because |\vec{A}(t)| = constant)$
$\Rightarrow 2\frac{d\vec{A}}{dt}.\vec{A} = 0$
Hence, option (c) is correct.

2. Solution:
$\vec{\nabla}h$ Gives the direction of steepest ascent
$\vec{\nabla}h\big|_{(0,0)} = e(-4\hat{\imath} + 8\hat{\jmath})$
Therefore, unit vector along the direction of steepest ascent is $\hat{n} = \frac{\vec{\nabla}h}{|\vec{\nabla}h|} = \frac{1}{\sqrt{80}}(-4\hat{\imath} + 8\hat{\jmath})$
Hence, option (a) is correct.

3. Solution:
Let, $\vec{A} = \frac{1}{\sqrt{2}}(1,1,0)$, $\vec{B} = \frac{1}{\sqrt{2}}(0,1,1)$ and $\vec{C} = \frac{1}{\sqrt{2}}(1,0,1)$

Now, $\vec{A}.(\vec{B} \times \vec{C}) = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} =$

$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} - 0\right) 0 \left(\frac{1}{\sqrt{2}} - 0\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} - 0\right) = \frac{1}{\sqrt{2}} \neq 0$
So, $\vec{A}, \vec{B}, \vec{C}$ are linearly independent.
Hence, option (b) is correct.

4. Solution: $Here, \vec{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$
$\therefore \quad \nabla^2\bar{A} = \frac{\partial^2}{\partial x^2}(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z) + \frac{\partial^2}{\partial y^2}(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z) + \frac{\partial^2}{\partial y^2}(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z) = 0$
Hence, option (c) is correct.

5. Solution:
Here, $\iint(\vec{\nabla} \times \vec{A}).\hat{n}.ds$
By using Divergence theorem
$= \iiint \vec{\nabla}.(\vec{\nabla} \times \vec{A})dV = 0 \qquad [\because \vec{\nabla}.(\vec{\nabla} \times \vec{A}) = 0]$
Hence, option (c) is correct.

6. Solution:

Here, $\iint_s \vec{r}.\hat{n}\,dS = \iiint \vec{\nabla}.\vec{r}\,dV$ (Using Divergence Theorem)

$= 3\iiint dV \quad as\ [\vec{\nabla}.\vec{r} = \vec{\nabla}.(x\hat{\imath} + y\hat{\jmath} + z\hat{k}) = 3]$

$\iint_s \vec{r}.\hat{n}\,dS = 3V$

Hence, option (d) is correct.

7. Solution:

We know that two vectors are orthogonal to if they are perpendicular to each other. It means the dot product of the two vectors should be zero if they are orthogonal.

In the above problem the given vector is $(\boldsymbol{a\hat{\imath}} + \boldsymbol{b\hat{\jmath}})$

By option (a), $(-b\hat{\imath} + a\hat{\jmath}).(a\hat{\imath} + b\hat{\jmath})$ = -ba-ab = 0

Hence matrix $(-b\hat{\imath} + a\hat{\jmath})$ will be orthogonal matrix to $(\boldsymbol{a\hat{\imath}} + \boldsymbol{b\hat{\jmath}})$.

Hence, option (a) is correct.

8. Solution:

Given vector $\varphi = 3x^2 + 4y - z$

$\vec{\nabla}\varphi = 6x\hat{\imath} + 4\hat{\jmath} - \hat{k}$

$|\vec{\nabla}\varphi|_{(1,1,7)} = \sqrt{36+16+1} = \sqrt{53}$

The unit vector normal to the surface,

$\left.\frac{\vec{\nabla}\varphi}{|\vec{\nabla}\varphi|}\right|_{(1,1,7)} = \frac{6\hat{\imath} + 4\hat{\jmath} - \hat{k}}{\sqrt{53}}$

Hence, option (c) is correct.

9. Solution:

Given vector $\vec{A} = (5x+2y)\hat{\imath} + (3y-z)\hat{\jmath} + (2x-az)\hat{k}$

Here $\vec{A}$ is solenoidal and we know that if a vector is solenoidal then there divergence will be zero.

So, $\vec{\nabla}.\vec{A} = 0$

$\Rightarrow$ 5 + 3 – a = 0

$\Rightarrow$ 8 – a = 0

$\Rightarrow$ a = 8

Hence, option (c) is correct.

10. Solution:

$\vec{\nabla} \times \vec{A} = i + j + k$

Hence, option (a) is correct.

11. Solution:

Given function $\phi = x^2y + xy$

$\vec{\nabla}\phi = (2xy + y)\hat{\imath} + (x^2 + x)\hat{\jmath}$

Now, $|\vec{\nabla}\phi| = \sqrt{(2xy+y)^2 + (x^2+x)^2}$

$= \sqrt{3^2 + 2^2}$ (at $x = y = 1$)

$= \sqrt{13}$

Hence, option (d) is correct.

12. Solution:

We know that the average value of a function on [a,b] is

$= \frac{1}{b-a}\int_a^b f(x)$

Given function is, $f(x) = \sin x$

The average value of a function in the interval $(0,\pi)$ is

$= \frac{1}{\pi}\int_0^\pi \sin x = \frac{1}{\pi}[-cos\pi]_0^\pi$

$\Rightarrow \quad = \frac{1}{\pi}$ [-(-1-1)]

$\Rightarrow \quad = \frac{2}{\pi}$

Hence, option (b) is correct.

13. Solution:

Unstable equilibrium is a state of equilibrium in which a small disturbance will produce a large change.

In the above graph we can easily find that the points r and u are the point of unstable equilibrium of the given potential.

Hence, option (c) is correct.

14. Solution:

$f(x) = 4x^3 (1 \le x \le 3)$

The average value of $f(x) = \frac{1}{(3-1)}\int_1^3 4x^3dx = \frac{1}{2}\int_1^3 4x^3dx = \frac{1}{2}\left[\frac{4x^4}{4}\right]_1^3 = \frac{1}{2}[3^4 - 1] = 40$

Hence, option (c) is correct.

15. Solution:

Unit normal vector to the curve : $\phi = x^3y^3 + xy - 17$ at point (2,0) is

$\hat{n} = [\nabla\phi|\nabla\phi]_{(2,0)} = \left.\frac{(3x^2y^2y)\hat{\imath} + (2x^3y + x)\hat{\jmath}}{|\nabla\phi|}\right|_{(2,0)} = \frac{2\hat{\jmath}}{2} = \hat{\jmath}$

Hence, option (d) is correct.

16. Solution:

$\bar{F} = x\hat{\imath} + 2y\hat{\jmath} + 3z\hat{k}, \nabla \times \bar{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 2y & 3z \end{vmatrix} = \hat{\imath}(0) - \hat{\jmath}(0) + \hat{k}(0) = 0$

$\bar{\nabla} \times \bar{F} = 0 \Rightarrow \bar{\nabla} \times (\bar{\nabla} \times \bar{F}) = 0$

Hence, option (a) is correct.

17. Solution:

$$\begin{bmatrix}1 & 1 & 0\\0 & 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}x_1+x_2\\x_2-x_3\end{bmatrix}$$

Hence, option (a) is correct.

18. Solution:

$\oint_s \frac{\vec{r}\cdot d\vec{S}}{r^3}=\int_V \vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right)dV$ (Using divergence theorem)

Since the closed surface encloses the origin i.e. $\vec{r}=0$

So, $\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right)=4\pi\delta^3(\vec{r})$

So, $\int_V \vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right)dV=4\pi\int_V \delta^3(\vec{r})dV=4\pi$

Hence, option (c) is correct.

19. Solution:

Statement (a) is true.

Hence, option (a) is correct.

20. Solution:

According to Stokes theorem,

$\oint \vec{r}\,.\overrightarrow{dr}$ = $\int\int(\vec{\nabla}\times\vec{r}).\overrightarrow{dS}$ = 0

($\because \vec{\nabla}\times\vec{r}=0$)

Hence, option (a) is correct.

21. Solution:

if $\vec{A}.(\vec{B}\times\vec{C})$=0

$[\vec{A}.\ \vec{B}.\vec{C}]=0$

$\vec{A}.\ \vec{B}.\vec{C}$ are coplanar.

Hence, option (a) is correct.

B. MATRICES

1. A square matrix A is unitary if:
(a) $A^\dagger=A$ (b) $A^\dagger=A^{-1}$
(c) $Tr(A)=1$ (d) $det\ (A)=1$

2. The eigenvalues of the matrix $M=\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$ are
(a) 1, 0 (b) 1, 1
(c) 1, 2 (d) 0,2

3. If two matrices A and B can be diagonalised simultaneously, which of the following is true?
(a) $A^2B=B^2A$
(b) $A^2B^2=B^2A$
(c) $AB=BA$
(d) $AB^2AB=BABA^2$

4. Which of the following is true for the matrix $\begin{bmatrix}4 & -1 & -4\\3 & 0 & -4\\3 & -1 & -3\end{bmatrix}$?
(a) It is an idempotent matrix
(b) It is nilpotent matrix of order 3
(c) It is an involuntary matrix
(d) None of the above

5. The trace of the matrix of 2×2 order matrix is 1 and determinant 1. Which of the following has to be true?
(a) One of the eigenvalue is 0
(b) One of the eigenvalue is 1
(c) Both of the eigenvalues are 1
(d) Neither of the eigenvalues are 1

6. The product *MN* of two Hermitian matrices *M* and *N* is anti-Hermitian. It follows that
(a) {M, N} = 0 (b) [M,N] = 0
(c) $M^+ = N$ (d) $M^+ = N^{-1}$

7. A 3×3 matrix has eigenvalues $0, 2+i$ and $2-i$. Which one of the following statements is correct?
(a) The matrix is Hermitian
(b) The matrix is unitary
(c) The inverse of the matrix exists
(d) The determinant of the matrix is zero

8. A real traceless 4×4 unitary matrix has two eigenvalues -1 and $+1$. The other eigenvalues are
(a) Zero and $+2$ (b) zero and $+1$
(c) Zero and $+2$ (d) -1 and $+1$

9. The eigenvalues of the matrix $\begin{bmatrix}i & -1\\1 & i\end{bmatrix}$ are
(a) 0, $2i$ (b) i, i
(c) -$2i, 4i$ (d) $i, -i$

10. The determinant of a 3×3 real symmetric matrix is 36. If two of its Eigen values are 2 and 3 then the third eigenvalue is
(a) 4 (b) 6
(c) 8 (d) 9

11. Eigen values of the matrix
$$\begin{pmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 2i & -2i & 0\end{pmatrix}$$ Are
(a) $-2,-1,1,2$ (b) $-1,1,0,2$
(c) $1,0,2,3$ (d) $-1,1,0,3$