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## MH – SET PHYSICS


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# MATHEMATICAL METHODS

CHAPTER 1

**MH - SET PHYSICS**

**(PREVIOUS YEAR EXAM QUESTIONS)**

## VECTOR ALGEBRA & VECTOR CALCULUS:

**[AUG-2011]**

1. The angle between vector $\hat{\imath}+\hat{\jmath}$ and $\hat{\jmath}+\hat{k}$ is (in radian):
(A) $\pi$ (B) $\frac{\pi}{2}$
(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

**[NOV-2011]**

2. If $\bar{A}.(\bar{B}\times\bar{C})=0$ in 3-dimensional space, then:
(A) $\bar{A},\bar{B},\bar{C}$ Are coplanar
(B) $\bar{A}$ is a null vector
(C) $\bar{A},\bar{B},\bar{C}$ span the whole 3-D space.
(D) $\bar{B}=0$

**[NOV-2011]**

3. The dimension of the subspace spanned by the real vectors:
$$\begin{bmatrix}1\\1\\0\\0\end{bmatrix},\begin{bmatrix}2\\2\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\\0\end{bmatrix},\begin{bmatrix}2\\0\\0\\3\end{bmatrix},\begin{bmatrix}1\\-2\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$
(A) 2 (B) 3
(C) 4 (D) 5

**[FEB-2013]**

4. What is the volume of a parallelepiped spanned by the vectors:
$(\hat{\imath}+\hat{\imath}), (\hat{\jmath}+\hat{k}), (\hat{k}+\hat{\imath})$?
(A) Zero (B) 1
(C) 2 (D) 3

**[FEB-2013]**

5. The value of $\nabla(r^2)$ is:
(A) $\hat{r}$ (B) $2\vec{r}$
(C) 2| r | (D) zero

**[FEB-2013]**

6. The value of $\nabla^2$ (1/r) is:
(A) -1/2 (B) -4$\pi\delta$(r)
(C) 4$\pi$ (D) zero

**[DEC 2013]**

7. $\bar{r}$ is the position vector of any point on the surface of a cube of side L. The surface integral.
$\iint_s \vec{r}\,.\,d\vec{s}$ is:
(A) 0 (B) ∞
(C) $3L^2$ (D) $3L^3$

**[DEC-2013]**

8. The value of $\nabla^2(r^2)$ is:
(A) 3 (B) 6
(C) 2r (D) zero

**[DEC-2013]**

9. The polar plot of the equation $r=a\theta$ represents:
(A) circle (B) spiral
(C) Gaussian (D) parabola

**[SEPT-2015]**

10. If $\vec{a}$ is a constant vector, then $\vec{\nabla}\cdot(\vec{a}\times\vec{r})$ is equal to:
(A) $a$ (B) 2a
(C) 0 (D) $a/2$

**[SEPT-2015]**

11. The volume of the parallelepiped with sides:
$\vec{a}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$.
$\vec{b}=4\hat{\imath}+5\hat{\jmath}+6\hat{k}$ And
$\vec{c}=7\hat{\imath}+8\hat{\jmath}+10\hat{k}$ Is:
(A) $2+\sqrt{3}$ (B) $\sqrt{2}$
(C) $\sqrt{3}$ (D) 3

**[SEPT-2015]**

12. The area of the triangle whose base is given by $\vec{a}=5\hat{\imath}-3\hat{\jmath}+4\hat{k}$ and $\vec{b}=\hat{\jmath}-\hat{k}$ is another side is
(A) $\sqrt{50}$ / 2 (B) $\sqrt{61}$ / 2
(C) $\sqrt{14}$ / 2 (D) $\sqrt{51}$ / 2

**[MAY-2016]**

13. $\nabla\left(\frac{1}{|\vec{r}|}\right)$ Is given by:
(A) $\frac{1}{r}\hat{r}$ (B) $\frac{1}{r^3}(\hat{\imath}+\hat{\jmath}+\hat{k})$
(C) $\frac{\vec{r}}{r^3}$ (D) $r(\hat{\imath}+\hat{\jmath}+\hat{k})$

**[MAY-2016]**

14. Which of the following defines a conservation force?
(A) $\vec{\nabla}.\vec{F}=0$ (B) $\vec{\nabla}\times\vec{F}=0$
(C) $\oint\vec{F}.\overrightarrow{dr}=0$ (D) $\frac{d\vec{F}}{dt}=0$

**[MAY-2016]**

15. Consider the three vectors:
$\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$
$\vec{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$ and
$\vec{c}=\hat{\imath}-\hat{\jmath}-\hat{k}$
Which of the following statements is true?

(A) $\vec{a}, \vec{b}, \vec{c}$ are linearly independent

(B) $\vec{a}, \vec{b}$, are linearly dependent

(C) $\vec{b}, \vec{c}$ are at right angles to each other

(D) $\vec{a}$, and $\vec{c}$ are parallel

**[MAY-2016]**

16. The position vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then $\nabla(r^2\vec{r})$is given by:

(A) 0 (B) $5r^2$

(C) $r^2$ (D) $3r^2$

**[APR-2017]**

17. The dimension of the vector space of n x n symmetric matrices is:

(A) $n^2$ - n (B) $\frac{n(n+1)}{2}$

(C) $n^2/2$ (D) n(n - 1)/2

**[APR-2017]**

18. Consider the vector space of polynomials of degree less than or equal to 5. This vector space has dimension:

(A) 5 (B) 6

(C) Infinity (D) 4

**[APR-2017]**

19. If $r = \sqrt{(x^2+y^2+z^2)}$, grad $r$ $(\vec{\nabla} r)$ is:

(A) $\vec{r}/r$ (B) 0

(C) $r$ (D) $\vec{r}$

**[JAN-2018]**

20. If $r^2 = x^2+y^2+z^2, grad\ r^n$ is:

(A) 0 (B) $r^{n-1}\vec{r}$

(C) $nr^{n-2}\vec{r}$ (D) $n(n-2)r^n\vec{r}$

**[JAN-2018]**

21. Let V be a 5-dimensional vector space and V1 and V2 be subspaces of V which are 3-dimensional each. Then the dimension of V1 ∧ V2 is:

(A) 3 (B) 0

(C) 1 (D) 2

**[JUNE-2019]**

22. If a vector field $\overline{F} = x\hat{\imath} + 2y\hat{\jmath}, 3z\hat{k}$, then $\overline{\nabla} \times (\overline{\nabla} \times F)$ is:

(A) Zero (B) $\hat{\jmath}$

(C) $2\hat{\jmath}$ (D) $3\hat{k}$

**[JUNE-2019]**

23. If $\overline{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_j$, then $\nabla^2\overline{A}$ will be:

(A) 1 (B) 3

(C) 0 (D) – 3

**[DEC-2020]**

24. Unit vector perpendicular to $\bar{A} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\bar{B} = 3\hat{\imath} + 4\hat{\jmath} - \hat{k}$ is:

(A) $\frac{-3\hat{\imath}+5\hat{\jmath}+11\hat{k}}{\sqrt{155}}$ (B) $\frac{\hat{\imath}-\hat{\jmath}+2\hat{k}}{\sqrt{6}}$

(C) $\frac{4\hat{\imath}-\hat{\jmath}-5\hat{k}}{\sqrt{42}}$ (D) $\frac{\hat{\imath}+2\hat{\jmath}-4\hat{k}}{\sqrt{21}}$

**[DEC-2020]**

25. If S is a closed surface enclosing a volume V and $\hat{n}$ is the unit vector normal to the surface and $\bar{r}$ is the position vector, then the value of the integral $\iint_S \hat{n}\, dS$ is:

(A) V (B) 2V

(C) 0 (D) 3V

**[SEPT-2021]**

26. Consider a vector $\overline{v} = \frac{\overline{r}}{r^3}$. The surface integral of this vector over the surface of a cube of side a and centred at the origin is :

(A) zero (B) $2\pi$

(C) $2\pi a^3$ (D) $4\pi$

**[SEPT-2021]**

27. If a force $\overline{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that :

(A) $\overline{\nabla} \times \overline{F} = 0$ (B) $\overline{\nabla} \cdot \overline{F} = 0$

(C) $\overline{\nabla} V = 0$ (D) $\nabla^2 V = 0$

**[MARCH-2023]**

28. Consider vectors $\vec{a} = \hat{\imath} + 5\hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} - 5\hat{\jmath} + \hat{k}$ and $\vec{c} = \hat{\imath} + \hat{k}$. Which of the following statements is true?

(A) $\vec{a}, \vec{b}, \vec{c}$ are linearly independent

(B) $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent

(C) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal

(D) $\vec{a}, \vec{b}, \vec{c}$ are normalized

**[MARCH-2023]**

29. Which of the following equations represents a conservative force ?

(A) $\overline{F} = (xy + z^2)\hat{\imath} + x^2\hat{\jmath} - 2xz\hat{k}$

(B) $\overline{F} = (2xy + z^2)\hat{\imath} + x^2\hat{\jmath} + 2xz\hat{k}$

(C) $\overline{F} = (xy - z^2)\hat{\imath} - x^2\hat{\jmath} + 2xz\hat{k}$

(D) $\overline{F} = (xy - z^2)\hat{\imath} + x^2\hat{\jmath} + 2xz\hat{k}$

**ANSWER KEY**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{\pi}{3}$ | A | D | C | B | D | D | B | B | B |
| **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| D | D | $-\frac{1}{r^3}\hat{r}$ | B | A | B | B | B | A | C |
| **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | |
| C | A | C | A | C | D | A | A | B | |

## SOLUTIONS: VECTOR ALGEBRA & VECTOR CALCULUS

1. **Solution:**

Let $\theta$ be the angle between

the vector $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

Dot product between the vector

$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$ --------------- (i)

Angle between the vectors $\vec{A} = \hat{\imath} + \hat{\jmath}$ And $\vec{B} = \hat{\jmath} + \hat{k}$

Putting the value of $\vec{A}$ & $\vec{B}$ in equation (i)

$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{1^2+1^2}} \cdot \frac{(\hat{\jmath}+\hat{k})}{\sqrt{1^2+1^2}} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

No option is correct.

**2. Solution: (A)**

if $\vec{A}.(\vec{B} \times \vec{C})$=0

$[\vec{A}.\ \vec{B}.\vec{C}] = 0$

$\vec{A}.\ \vec{B}.\vec{C}$ are coplanar.

Hence option (A) is correct.

**3. Solution: (D)**

$$\begin{bmatrix}1\\1\\0\\0\end{bmatrix}\begin{bmatrix}2\\2\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\begin{bmatrix}2\\0\\0\\3\end{bmatrix}\begin{bmatrix}1\\-2\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

Independent vectors are spanned by the real vector i. e 5

Hence the correct option is (D)

**4. Solution: (C)** volume of parallelopiped is

$\vec{a}\ .\ \vec{b} \times \vec{c}$ = $[\vec{a}\ \vec{b}\ \vec{c}]$

Let $\vec{a} = \hat{\imath} + \hat{\jmath}$ , $\vec{b} = \hat{\jmath} + \hat{k}$ , c = $\hat{k} + \hat{c}$

$\vec{b} \times \vec{c} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\0 & 1 & 1\\1 & 0 & 1\end{vmatrix}$

= $\hat{\imath}$ [1-0]- $\hat{\jmath}$ [0-1] + $\hat{k}$[0-1]

= $\hat{\imath} + \hat{\jmath} - \hat{k}$

$\vec{a}\ .\ (\vec{b} \times \vec{c})$ = $(\hat{\imath}+\hat{\jmath})\ .\ (\hat{\imath} + \hat{\jmath} - \hat{k})$

= 1 + 1

= 2

Hence option (C) is correct

**5. Solution: (B)**

$$\vec{\nabla} \cdot (r^2) = 2r\hat{r}$$
$$\vec{\nabla} \cdot (r^2) = 2r\hat{r}$$
$$\hat{r} = \frac{\vec{r}}{|r|} \Rightarrow r\hat{r} = \vec{\vec{r}}$$
$$\vec{\nabla}(r^2) = \hat{r}\frac{\partial}{\partial r}(r^2) = 2r\hat{r}$$
$$\vec{\nabla}(r^2) = 2\vec{r}$$

Hence option (B) is correct.

**6. Solution: (D)**

$\vec{\nabla}^2\left(\frac{1}{r}\right) \Rightarrow \vec{\nabla} \cdot \vec{\nabla}\left(\frac{1}{r}\right) \Rightarrow \vec{\nabla} \cdot \hat{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\right)$

$\vec{\nabla} \cdot \left(-\frac{1}{r^2}\right)\hat{r} \Rightarrow \vec{\nabla} \cdot \left(\frac{-\vec{r}}{r^3}\right)$

$\nabla(\phi \cdot \vec{A}) = \phi(\vec{\nabla} \cdot \vec{A}) + \vec{A}(\vec{\nabla} \cdot \hat{\phi})$

$\phi$-scalar field, $\vec{A}$ = vector field.

$$\nabla \cdot \left(\frac{-r}{r^3}\right) = \frac{-1}{r^3}(\vec{\nabla} \cdot \vec{r}) - \vec{r} \cdot \nabla\left(\frac{1}{r^3}\right)^+$$
$$= \frac{-3}{r^3} - \vec{r} \cdot \left(\frac{-3}{r^4}\right) \cdot \hat{r}$$
$$= -\frac{-3}{r^3} + \frac{3}{r^3}$$
$$\Rightarrow \nabla^2\left(\frac{1}{r}\right)^4 = 0$$

Hence option (D) is correct.

**7. Solution: (D)**

Since $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\bar{\nabla} \cdot \bar{\gamma} = 3$

By Gauss-Divergence theorem,

$$\iint_s \cdot \bar{r} \cdot d\bar{s} = \iiint_V (\bar{\nabla} \cdot \bar{r})d\tau$$
$$= 3\iiint_v d\tau$$
$$\iint_s \bar{r} \cdot d\bar{s} = 3L^3$$

Hence option (D) is correct

**8. Solution: (B)**

$\nabla^2(x^2 + y^2 + z^4) = \left(\frac{\partial^2}{\partial x+} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z}\right) \cdot (x^2 + y^2 + z^2) = 6$

Hence option (B) is correct

**9. Solution: (B)**

The graph of the equation $r = \text{a}\theta$ is spiral path.

Hence option (B) is correct

**10. Solution: (B)**

$\vec{a}$ = a$(\hat{\imath} + \hat{\jmath} + \hat{k})$

$\vec{r}$ = x$\hat{\imath}$ + y$\hat{\jmath}$ + z$\hat{k}$

$\vec{a} < \vec{r}$ = a $\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\1 & 1 & 1\\x & y & z\end{vmatrix}$

= a [$\hat{\imath}$ (z - y)) + $\hat{\jmath}$(x - z) + $\hat{k}$ (y -x)]

$\nabla \times (\vec{a} \times \vec{r})$ = a $\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\\partial/\partial x & \partial/\partial y & \partial/\partial y\\z-y & x-y & y-x\end{vmatrix}$

= a[i(1-(-1)- $\hat{\jmath}$(-1-1)+ $\hat{k}$ (2))]

= 2a$\hat{\imath}$ + 2a$\hat{\jmath}$ + 2a$\hat{k}$

= 2a

Hence option (B) is correct

**11. Solution: (D)**

$\vec{a}$ = I + 2j + 3$\hat{k}$

$\vec{b}$ = 4i + 5 j + 6$\hat{k}$

$\vec{c}$ = 7i + 8j + 10$\hat{k}$

$\vec{a} \times \vec{b} \times \vec{c} = \begin{vmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 10\end{vmatrix}$

$|\vec{a} \times \vec{b} \times \vec{c}\ |$ = 1(50 - 48) -2(40 - 42) + 3(32-35)

= 1[2] – 2[-2] + 3 [-3]

= 2 + 4 – 9

$|\vec{a}\times\vec{b}\times\vec{c}\,|$ = -3

|volume|= |-3|

volume = 3

Hence option (D) is correct

**12. Solution: (D)**

Given $\vec{a}$ = $5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\vec{b}$ = $\hat{\jmath}$ - $\hat{k}$

Area of trangle = $\frac{1}{2}$ bh

$$|\vec{a}\times\vec{b}| = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & -3 & 4 \\ 0 & 1 & -1 \end{vmatrix}$$

$|\vec{a}\times\vec{b}|$ = $\hat{\imath}$ [(-1 × −3) – (1 × 4)] – j[(-1 × 5)- (0 × 4)] + E[(1 × 5) – (0x - 3)]

= $\hat{\imath}$ [3 - 4]- $\hat{\jmath}$ [-5] + $\hat{k}$ [5]

$|\vec{a}\times\vec{b}|$ = $-\hat{\imath} + 5\hat{\jmath} + 5\hat{k}$

$= \sqrt{(-1)^2 + (5)^2 + (5)^2}$

$= \sqrt{1^2 + 5^2 + 5^2}$

$\vec{a}\times\vec{b} = \sqrt{51}$ Area of trangle $\frac{1}{2}$ 6h

$= \frac{\sqrt{51}}{2}$

Hence option (D) is correct

**13. Solution:**

$$\nabla\frac{1}{|\vec{r}|} = \hat{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\right)$$

$$= \frac{-1}{r^2}\hat{r}$$

Short cut $\nabla r^n = nr^{n-2}\vec{r}$

$$\nabla\left(\frac{1}{r}\right) = (-1)(r)^{(-1-2)}\cdot\vec{r}$$

$$\nabla\left(\frac{1}{r}\right) = -\frac{1}{r^3}\hat{r}$$

No option is correct.

**14. Solution: (B)**

If a force is conservative

$$\int \vec{F}\cdot\vec{dl} = 0$$

By using stoke's theorem

$$\int \vec{F}\cdot\vec{dl} = \iint (\vec{\nabla}\times\vec{F})ds = 0$$

Hence option (B) is correct.

**15. Solution: (A)**

$\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}.$

$\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\vec{c} = \hat{\imath} - \hat{\jmath} - \hat{k}$

3 vectors are given

linearly dependent linearly independent

if | det |= 0 if |det l ≠ 0

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{vmatrix}$$

$\Rightarrow 1(1+1) - 1(-1-1) + 1(-1+1)$

$\Rightarrow 2 + 2 + 0$

$\Rightarrow 4 \neq 0$

So, $\vec{a}, \vec{b}, \vec{c}$ are linearly independent,

Hence option (A) is, correct.

**16. Solution: (B)**

$\nabla\,(r^2\,r)$

The position vector r = $x\hat{\imath}$ + $y\hat{\jmath}$ + $z\hat{k}$

$r^2$ = $(X^2 + Y^2 + Z^2)$

Now calculate $r^2.r$

$r^2$.r = $(x^2 + y^2 + z^2)$($x\hat{\imath}$ + $y\hat{\jmath}$ + $z\hat{k}$)

Now take the divergence (∇.) of this expression

∇. ($r^2$.r) = 3($x^2 + y^2 + z^2$) + 2(x + y + z)

= 3$r^2$ + 2r

= 5$r^2$

Hence Option is B is correct

**17. Solution: (B)**

The dimension of vector space of $n \times n$ symmetric matrices is $\frac{n(n+1)}{2}$

Hence , option (B) is correct.

**18. Solution: (B)**

Dimension of a vector spaces= Highest degree of polynomial + 1 Dimension of a vector space =5+1=6

Hence, option (B) is correct.

**19. Solution: (A)**

Let $|\vec{x}| = \sqrt{x^2 + y^2 + z^2}$

$\hat{x}\frac{\partial}{\partial x}\sqrt{x^2 + y^2 + z^2}$ this can be computed with the chain rule

Let $\frac{\partial}{\partial x}$ g[h(x)], h(x) = $x^2 + y^2 + z^2 \;\Rightarrow\; \frac{\partial h}{\partial x} = 2x$

g(h) = $\sqrt{h} \;\rightarrow\; \frac{\partial g}{\partial h} = \frac{\partial\,(h)^{\frac{1}{2}}}{\partial h} = \frac{1}{2}\,h^{-\frac{1}{2}}$

$$\frac{\partial}{\partial x}\sqrt{x^2 + y^2 + z^2} = 2x \times \frac{1}{2}\frac{1}{\sqrt{x^2+y^2+z^2}} = \frac{x}{\sqrt{x^2+y^2+z^2}}\,\hat{x}$$

$$\hat{y}\frac{\partial}{\partial y}\sqrt{x^2 + y^2 + z^2} = \frac{y}{\sqrt{x^2+y^2+z^2}}\,\hat{y}$$

$$\hat{z}\frac{\partial}{\partial y}\sqrt{x^2 + y^2 + z^2} = \frac{z}{\sqrt{x^2+y^2+z^2}}\,\hat{z}$$

$$\nabla.f = \hat{x}\frac{x}{\sqrt{x^2+y^2+z^2}} + \hat{y}\frac{y}{\sqrt{x^2+y^2+z^2}} + \hat{z}\frac{z}{\sqrt{x^2+y^2+z^2}}$$

With $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \hat{x}.x + \hat{y}.y + \hat{z}.z$

$\sqrt{r^2} = \sqrt{x^2+y^2+z^2} = |\vec{r}|$

$\vec{\nabla}f = \frac{\vec{r}}{|r|}$

Hence option (A) is correct

**20. Solution: (C)**

Here $r^2 = x^2 + y^2 + z^2$

gives

$$2r\frac{\partial r}{\partial x} = 2x$$

$$2r\frac{\partial r}{\partial y} = 2y$$

$$2r\frac{\partial r}{\partial z} = 2z$$

$$\frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

Now $r^n \Rightarrow \frac{\partial v}{\partial x} = nr^{n-1}$

$$\frac{\partial r}{\partial x} = nr^{n-1}\left(\frac{x}{r}\right)$$

$$= nr^{n-2}x$$

$$x = \vec{r} = (xi + 4j + 2k)$$

$$= nr^{n-2}\vec{r}$$

option C

**or**

$$\nabla(r^n) = \sum \hat{\imath}\frac{\partial}{\partial x}(r^n) = \sum \overrightarrow{\imath n}^{n-1}\frac{\partial r}{\partial x}$$

$$= \sum \overrightarrow{\imath n}^{n-1}\frac{x}{r}$$

$= nr^{n-1}$ vector $(\text{xi} + yj + zk)$

$= nr^{n-2}\,\vec{r}$

Hence "option " (C) is correct

**21. Solution: (C)**

$v \rightarrow 5$ dimensional vector space be

Let $v = [a, b, c, d, e]$

$v_1$ and $v_2$ be subspace of $v$ having

3 -dimensional each.

Let
$$v_1 = [a, b, c]$$
$$v_2 = [c \quad d \quad e]$$
$$\therefore\ v_1 \cap v_2 = [c]$$
$$= 1$$

**or**

$$v = v_1 + v_2 - v_1 n\, v_2$$

$$5 = 3 + 3 - v_1 \cap v_2$$

$$v_1 \cap v_2 = 6 - 5$$

$$v_1 \cap v_2 = 1$$

Hence option (C) is Correct

**22. Solution: (A)**

$$\vec{F} = x\hat{\imath} + 2y\hat{\jmath} + 3z\hat{k}$$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 2y & 3z \end{vmatrix} = 0$$

$$\Rightarrow \hat{\imath}\left[\frac{\partial}{\partial y}(3z) - \frac{\partial}{\partial z}(2y)\right] - \hat{\jmath}\left[\frac{\partial}{\partial x}(3z) - \frac{\partial}{\partial z}(x)\right] +$$
$$\hat{k}\left[\frac{\partial}{\partial x}(2y) - \frac{\partial}{\partial y}(x)\right] = 0$$

$$\Rightarrow\ 0 + 0 + 0 = 0$$

Hence, option (A) is correct.

**23. Solution: (C)**

$\vec{\nabla}^2 A = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}\times(\vec{\nabla}\times\vec{A})$, in solving for curl of $\vec{A}$ and divergence of $\vec{A}$, the answer comes out to be zero.

Hence, option (C) is current.

**24. Solution: (A)**

Given vector are

$\vec{A}$ = 2i – j + $\hat{k}$

$\vec{B}$ = 3i + 4j – k

This can be found by cross product

$$\vec{A}\times\vec{B}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$$

$\vec{A}\times\vec{B}$ = i[1 + 4] -j [-2 + 3] + k[8 + 3]

= i[5] – j [1] + k[5]

$\vec{A}\times\vec{B}$ = -3i + 5j + 11$\hat{k}$

A unit vector along $\vec{A}\times\vec{B} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

$= \frac{-3\hat{\imath}+5\hat{\jmath}+11\hat{k}}{\sqrt{(-3)^2+(5)^2+(11)^2}}$

$\vec{A}\times\vec{B} = \frac{-3\hat{\imath}+5\hat{\jmath}+11\hat{k}}{\sqrt{155}}$

Hence option (A) is correct

**25. Solution: (C)**

The integral $\int \hat{u}$ds over a closed surface s is related to the volume V by the volume v by the divergence theorem which states that $\int\int\int v(\nabla.\hat{u})$ dv equals $\int\int s\hat{u}$.ds if $\hat{u}$ is constant vector then $(\nabla.\hat{u})$ is zero and the integral over s is zero [the $\hat{u} = \hat{n}$]

Therefore the correct option is (C)

**26. Solution: (D)**

Using Gauss divergence theorem we get

$= 4\pi\delta^3(r-0)$

$= 4\pi(1)$ since $[\delta^3(r-0) = 1]$

$= 4\pi$

Hence option (D) is correct.

**27. Solution: (A)**

We know that $f = \frac{-dv}{dr}$, where $r$ is the distance from the origin of the coordinate system.

⇒ curl of $F = 0$

i.e. $\vec{\nabla} \times \vec{F} = 0$

Hence, option (A) is correct.

**28. Solution: (A)**

$\vec{a} = \hat{\imath} + 5\hat{\jmath} + \hat{k}$

$\vec{b} = \hat{\imath} - 5\hat{\jmath} + \hat{k}$

$\vec{c} = \hat{\imath} + \hat{k}$

$$\vec{a} \times \vec{b} \times \vec{c} = \begin{vmatrix} 1 & 5 & 1 \\ 1 & -5 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$= -5 - 10 + 5$

$= -15 + 5$

$= -10 \neq 0$

The determent of vectors is not equal to zero. it means vectors are linearly independent

[If the vectors are equal to zero then it consider as linearly dependent]

Hence option (A) is Correct

**29. Solution: (B)**

For conservation force $\nabla \times \bar{F} = 0$

From option B,

$$\vec{\nabla} \times \bar{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2xy + z^2 & x^2 & 2xz \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = 0$$

Hence option (B) is correct

## MATRICES

**[AUG-2011]**

1. The real matrix $A = \begin{bmatrix} a & f & g \\ -f & a & -h \\ -g & h & a \end{bmatrix}$ is skew symmetric when:

(A) $a = 0$ (B) $f = 0$

(C) $g = h$ (D) $f = g$

**[AUG-2011]**

2. The eigenvalues of the matrix $\begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix}$ by:

(A) 1 (B) $\pm\omega$

(C) $\pm\omega^2$ (D) $\pm i$

**[AUG-2011]**

3. The determinant of a $3 \times 3$ real symmetric matrix is 36. If two of its eigenvalues are 2 and 3, then the sum of the eigenvalues is:

(A) 30 (B) 10

(C) 11 (D) 31

**[NOV-2011]**

4. The eigenvalues of the matrix $\begin{bmatrix} i & -i & 0 \\ 0 & 1 & i \\ 0 & 0 & -i \end{bmatrix}$ Are:

(A) $i, -i, 0$ (B) $i, i^2, i^3$

(C) $1, 0, -1$ (D) $1, i, -i$

**[FEB-2013]**

5. The eigenvalues of (2 × 2) matrix are:

$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$

(A) $0, 2$ (B) $1, 3$

(C) $3, -1$ (D) $1 + i, 1 - i$

**[FEB-2013]**

6. The trace of an antisymmetric matrix is:

(A) Real (B) Zero

(C) Pure Imaginary (D) Unity

**[FEB-2013]**

7. If A is an antisymmetric matrix of odd order n, then the determinant of A is :

(A) positive real number (B) negative real number

(C) zero (D) real number $(-1)^{(n+1)/2}$

**[DEC 2013]**

8. A (3 × 3) matrix has unequal eigenvalues a, b and c and its determinant is D. If a = –1, b = 2 and D = 4, what is the value of c?

(A) 1 (B) –1

(C) –2 (D) 0

**[DEC 2013]**

9. A matrix $M$ is of the form $M = \begin{bmatrix} 0 & a \\ -a^* & 0 \end{bmatrix}$

If the $det\ M = 1$, the most general value of 'a' is:

(A) Cos θ

(B) sin θ

(C) $exp(i\theta)$

(D) cosh θ where θ is a real parameter.

**[SEPT-2015]**

10. A, B, C, D are $(n \times n)$ matrices, each with non-zero determinant. If:

ABCD = I, then $B^{-1}$ is

(A) $D^{-1}C^{-1}A^{-1}$ (B) $CDA$

(C) ADC (D) Does not exist

**[SEPT-2015]**

11. The eigenvalues of the matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$ are:

(A) 1 , 4, 3 (B) 3, 7, 3

(C) 7, 3, 2 (D) 1, 2 ,3

**[MAY-2016]**

12. Eigenvalues of the matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ are:

(A) $1\ , -1$ (B) $-1\ , \ -i$

(C) $i, \ -i$ (D) $1 + i, 1 - i$