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Mathematical Methods in Physics

Unit 1.1: Dimensional Analysis

- Using dimensional analysis, find the correct option corresponding to pressure (where h is Planck's constant, v is velocity and x is displacement)
 - hv / x^4
 - $h^{0.5}v / x^4$
 - $h^{0.5}v / x^6$
 - $h^{0.5}v^2 / x^4$
- From the given options, choose the pair in which the physical quantities have identical dimensions:
 - Young's modulus and Planck's constant
 - Angular momentum and Impulse
 - Young's modulus and Impulse
 - Pressure and Young's Modulus
- If the velocity v , force F and area A are taken as the base units, then which of the following options corresponds to the mass density:
 - $F / A v^2$
 - Fv / A^3
 - F / A^2v
 - Av^2 / F^2
- Using dimensional analysis, find the correct corresponding to the gravitational constant G (where $k_b =$ Boltzmann constant, $T =$ temperature, $h =$ Planck's constant, $m =$ mass and $c =$ speed of light)
 - hc^2 / m
 - $hc^5 / k_b T$
 - $k_b T^2 / h^2 c^3$
 - hc / m^2
- Using dimensional analysis, find the correct option corresponding to moment of inertia (where J is impulse, v is velocity and A is area)
 - $J / A v^2$
 - $J A / v$
 - $J A^2 / v$
 - Av^2 / J^2

ANSWER KEY

1	2	3	4	5
a	d	a	d	b

:: Solutions ::

1. Since, dimensions of pressure is given by :-

Force = pressure \times Area = mass acceleration.

$$\text{or } p = \frac{\text{mass} \times \text{acceleration}}{\text{area}} = ML^{-1}T^{-2}$$

and h (Planck's constant) has the dimension of angular momentum i.e., ML^2T^{-1}

and velocity as: LT^{-1}

and displacement as: L

Clearly, from options, $\frac{hv}{x^4}$ will give $\frac{ML^2T^{-1} \cdot LT^{-1}}{L^4}$ or $ML^{-1}T^{-2}$

which is the same as the dimensions of pressure.

\therefore option (a) is correct.

\therefore option (b) is correct.

2. Since the dimension of Planck's constant is same as that of angular momentum i.e. ML^2T^{-1} and the dimension of

Young's Modulus i.e. $\frac{F}{A} \cdot \frac{\Delta l}{l}$ is $ML^{-1}T^{-2}$

$$\text{Impulse} = \int F dt = MLT^{-1}$$

$$\text{Pressure} = \frac{F}{A} = ML^{-1}T^{-2}$$

\therefore option (d) is correct.

3. Since, mass density i.e. $\frac{\text{Mass}}{\text{Volume}}$ has dimension ML^{-3}

Also, the dimension of: -force is MLT^{-2}

Area (A) is L^2

Velocity (v) is LT^{-1}

$$\text{So, clearly } \frac{F}{Av^2} = \frac{MLT^{-2}}{L^2 \cdot L^2 T^{-2}} = ML^{-3}$$

\therefore option (a) is correct.

4. Since Energy of different forms can be written as $= \frac{1}{2} mv^2$
 $= k_b T$

$$= \frac{-GMm}{r}$$

$$\therefore G = \frac{(\text{Energy}) \cdot L}{M^2} \text{ gives } ML^2T^{-2} \cdot L \cdot M^{-2} \\ = M^{-1}L^3T^{-2} \quad - (1)$$

Also, we know Planck's constant (h) has dimensions ML^2T^{-1}

and c has dimension LT^{-1}

$$\therefore \frac{hc}{m^2} = \frac{ML^2T^{-1} \cdot LT^{-1}}{M^2} \\ = M^{-1}L^3T^{-2} \quad - (2)$$

\therefore from (1) & (2), option (d) is correct.

5. Since we know that the dimensions of moment of inertia is ML^2 & that of velocity is LT^{-1}

Area is L^2

Impulse is MLT^{-1}

So, clearly, $\frac{\text{Impulse}}{\text{Velocity}}$ gives M as the resulting dimension.

So, by multiplying $\frac{\text{Impulse}}{\text{Velocity}}$ with area, we will have the same dimensions as that of moment of inertia i.e. ML^2

Unit 1.2: Vector Algebra

- A unit vector \hat{n} in x-y plane is at angle 150° w.r.t. \hat{i} . The angle between the vectors $\vec{u} = a\hat{i} + b\hat{n}$ and $\vec{v} = b\hat{i} + a\hat{n}$ will be 120° . Then $a^2 + b^2$ will approximately be
 - $3ab$
 - $2ab$
 - ab
 - $4ab$
- Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If \vec{c} is such that $\vec{a} \cdot \vec{c} = c$ and $|\vec{c} - \vec{a}| = 2$, then the value of $|c|$ will be
 - $1 \pm \sqrt{2}$
 - 0
 - $\sqrt{2}$
 - $\sqrt{3}$
- Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If \vec{c} is such that $\vec{a} \times \vec{c} = c$ and $|\vec{c} - \vec{a}| = 2$ and the angle between $(\vec{a} \times \vec{b})$ and \vec{c} is 30° . Then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ will be
 - $\frac{1-\sqrt{3}}{2}$
 - $\frac{1+\sqrt{2}}{\sqrt{2}}$
 - $\frac{\sqrt{3}}{2}$
 - 0
- Given that for three vectors \vec{a}, \vec{b} and \vec{c} , $\vec{a} \cdot (\vec{b} \times \vec{c}) = 4$, then what will be $\vec{c} \cdot (\vec{a} \times \vec{b})$
 - 2
 - 4
 - 8
 - 12
- Given that $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$, then $\vec{a} \times \vec{b} \times \vec{c}$ will be
 - $-\hat{i} + \hat{j}$
 - $\hat{i} + \hat{j}$
 - $\hat{i} + \hat{j} + \hat{k}$
 - $\hat{j} + \hat{k}$
- Given that $\vec{a} = \hat{i} + \hat{j}$, $\vec{a} \cdot \vec{c} = \vec{a} \cdot \vec{b}$ and $\vec{b} = \vec{c}$, then $\vec{a} \times \vec{b} \times \vec{c} =$
 - 4
 - 3
 - 6
 - 0
- Consider three-unit vectors \vec{a}, \vec{b} and \vec{c} such that $\vec{a} \times \vec{b} \times \vec{c} = \frac{\vec{b} - \vec{c}}{2}$, then what will be the angle (in degrees) that \vec{a} makes with \vec{b}
 - 45
 - 30
 - 60
 - 90
- Consider three-unit vectors \vec{a}, \vec{b} and \vec{c} such that $\vec{a} \times \vec{b} \times \vec{c} = \frac{\vec{b}}{2} - m\vec{c}$, what will be the value of m such that the angle between \vec{a} and \vec{b} is 120°
 - $\frac{1}{3}$
 - $\frac{1}{2}$
 - $\frac{1}{9}$
 - -2
- Consider that the equation of a surface is given by $x^2 + 2y^2 + 3z^2 = 0$, then the unit normal at the point $(1, 1, 0)$ is
 - $\frac{2\hat{i} + \hat{j}}{\sqrt{3}}$
 - 0
 - $\frac{\hat{i} - 2\hat{j}}{\sqrt{3}}$
 - $\frac{\hat{i} + 2\hat{j}}{\sqrt{5}}$
- Given that the two surfaces $ax^2 + by^2 + z^2 = 0$ and $2ax^2 - by^2 = 0$ at the point $(1, 1, 1)$ are orthogonal to each other. Find the relation between a and b
 - $a = b$
 - $a = 2b$
 - $b = 2a$
 - $b = \sqrt{2}a$
- Given that the equation of surface is $x^2 + y^2 + z^2 = 0$, then equation of tangent plane to the surface at $(1, 2, 1)$ will be
 - $x + y + z = 4$
 - $x + 2y + z = 6$
 - $x + y + z = 0$
 - $x + y = 6$
- Given that the equation of surface is $x^2 y z = 2$, then equation of tangent plane to the surface at $(1, 1, 1)$ will be
 - $x + 2y + z = 4$
 - $2x + y + z = 4$
 - $x + 2y + 4z = 6$
 - $x + y = 0$
- Find the value of directional derivative of the surface $x^2 + y^2 + z^2 = 0$ in the direction of $\vec{A} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$ at $(1, 1, 0)$
 - $\sqrt{3} + 1$
 - $\sqrt{3}$
 - 0
 - 1
- The directional derivative of scalar function $\Phi(x, y) = \frac{x}{x^2 + y^2}$ along a line makes 60° angle with positive x-axis at $(1, 1)$ is
 - $\frac{\sqrt{3}}{2}$
 - $-\frac{\sqrt{3}}{2}$
 - 2
 - -2
- Find the angle (in degrees) w.r.t. the tangent of the surface at which the direction along which the directional derivative is 0
 - 45
 - 60
 - 30
 - 90
- Find the angle (in degrees) w.r.t. the tangent of the surface at which the direction along which the directional derivative is maximum
 - 0
 - 60
 - 45
 - 75
- Which among the following tells us the rate of change of the surface/scalar function perpendicular to the surface itself
 - Divergence
 - Curl
 - Gradient
 - None of these
- The divergence of a vector function tells us about
 - Net outward flux
 - Rotational Effect
 - both a and b
 - none of these
- For a vector field $\vec{A} = \hat{r} r^2$, then divergence of \vec{A} will be
 - $9r^2$
 - $3r^2$
 - $5r^2$
 - $2r^2$

20. for a constant vector field \vec{a} , the value of $\nabla \times \vec{a} \times \vec{r}$ will be
 (a) $4\vec{a}$ (b) 0
 (c) \vec{a} (d) $2\vec{a}$
21. for a constant vector field \vec{a} , the value of $\nabla \cdot (\vec{a} \cdot \vec{r})$ will be
 (a) $4\vec{a}$ (b) 0
 (c) \vec{a} (d) $2\vec{a}$
22. Consider a conservative field \vec{A} . What will be the work done by the conservative field along a closed curve $x^2 + y^2 = 1$
 (a) 0 (b) -2
 (c) 2 (d) $\frac{1}{2}$
23. For a vector field $\vec{A} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$, what will be the work done by \vec{A} over a square having sides 1 unit and centre at (1,1)
 (a) 13 (b) 9
 (c) 0 (d) -25
24. For a vector field $\vec{F} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$, what will be the work done by \vec{F} over a curve $x^2 + y^2 = 12$
 (a) 0 (b) π
 (c) 2π (d) $-\pi$
25. The vector field $\vec{A} = -(x^2\hat{i} + y\hat{j})$ is:
 (a) Solenoidal
 (b) Irrotational
 (c) both a and b
 (d) none of these
26. $\vec{A} = (-mx\hat{i} + y\hat{j})$, for what value of m, will be the \vec{A} solenoidal
 (a) 0 (b) 1
 (c) -2 (d) 4
27. What should be the value of m, such that the vector field $\vec{A} = (-mx\hat{i} + y\hat{j})$ in the region $(x-2)^2 + (y-2)^2 = 1$, is irrotational
 (a) 2 (b) 1
 (c) 0 (d) -1
28. For the constant vector \vec{a} , the force field is defined as $\vec{F} = \vec{r} \times \vec{a}$, find the work done by the force field over the closed loop $x^2 + y^2 = 1$
 (a) $2\pi a$ (b) $-2\pi a$
 (c) 2π (d) -0
29. Find the work done by the vector field $\vec{A} = ((xy + y^2)\hat{i} + x^2\hat{j})$
 Over the curve between $y=x$ and $y^2 = 4ax$
 (a) $7a^2$ (b) $6.4a^3$
 (c) $2\pi a^2$ (d) 0
30. Given that $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$, surface of cylinder: $x^2 + y^2 = 4, z = 0$ to $z = 3$. Then find the flux through this surface
 (a) 0 (b) 20π
 (c) 84π (d) 60π

ANSWER KEY

1	2	3	4	5	6	7	8	9	10
a	a	b	b	a	d	c	b	d	d
11	12	13	14	15	16	17	18	19	20
b	c	a	b	d	a	a	a	c	d
21	22	23	24	25	26	27	28	29	30
c	a	c	c	b	b	d	b	b	c

:: Solutions ::

1. $\vec{u} \cdot \vec{v} = uv \cos \cos 120^\circ = uv \left(\frac{-1}{2}\right)$
 Or $(a\hat{i} + b\hat{n}), (a\hat{n} + b\hat{i}) = \frac{1}{2}uv$
 Or $a^2 \cos \cos 150^\circ + ab + ab + b^2 \cos \cos 150^\circ =$
 $\sqrt{a^2 + b^2 + 2ab \cos \cos 150^\circ}$
 $\sqrt{a^2 + b^2 + 2ab \cos \cos 150^\circ}$
 Or $(a^2 + b^2)\sqrt{3} - 4ab = a^2 + b^2 = \sqrt{3}ab$
 Or $(a^2 + b^2) = ab \cdot \left(\frac{4-\sqrt{3}}{\sqrt{3}-1}\right) \approx 3ab$
2. Given $\vec{a} \cdot \vec{c} = 0$ or $ac \cos \cos \phi = c$
 Or $a \cos \cos \phi = 1$ ---- (1)
 & $|\vec{c} - \vec{a}| = 2$ or $\sqrt{a^2 + c^2 - 2ac \cos \cos \theta} = 2$
 Using (1), $\sqrt{a^2 + c^2 - 2c} = 2$
 Or $a^2 + c^2 - 2c = 4$
 Where $|a|^2 = 3$
 Or $c = 1 + \sqrt{2}$
3. $|\vec{a} \times \vec{b} \times \vec{c}| = |\vec{a} \times \vec{b}| |\vec{c}| \sin 30^\circ$
 $= |-i + j| |\vec{c}| \sin 30^\circ$
 $= (\sqrt{2})(1 \pm \sqrt{2}) \frac{1}{2}$
 $= \frac{1 \pm \sqrt{2}}{\sqrt{2}}$
4. Since we know that the scalar triple product of 3 vector $\vec{a}, \vec{b}, \vec{c}$ are
 $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$ and
 given: $\vec{a} \cdot (\vec{b} \times \vec{c}) = 4$
 $\therefore \vec{c} \cdot (\vec{a} \times \vec{b}) = 4$
5. Since we know that: $a \times b \times c = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$
 $= (\hat{j} + \hat{k})(1) - (\hat{i} + \hat{k})(1)$
 $= \hat{j} = \hat{i}$
6. Since $\vec{a} \times \vec{b} \times \vec{c} = \vec{b} \cdot (\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$
 & given that $\vec{a} \cdot \vec{c} = \vec{a} \cdot \vec{b}$ & $\vec{b} = \vec{c}$
 $\vec{a} \times \vec{b} \times \vec{c} = \vec{b} \cdot (\vec{a} \cdot \vec{c}) - (\vec{b} \cdot \vec{c})$
 $= 0$
7. Since $\vec{a} \times \vec{b} \times \vec{c} = \vec{b} \cdot (\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$
 Given: $\vec{a} \times \vec{b} \times \vec{c} = \frac{\vec{b}}{2} - \frac{\vec{c}}{2}$
 Or $\vec{a} \cdot \vec{b} = \frac{1}{2}$
 $ab \cos \cos \theta = \frac{1}{2}$
 & given that $|a| = |b| = 1 \therefore \theta = 60^\circ$
8. Given $\vec{a} \times \vec{b} \times \vec{c} = \frac{\vec{b}}{2} + m\vec{c}$
 Since $\vec{a} \times \vec{b} \times \vec{c} = \vec{b} \cdot (\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$
 On comparing, we have: $-(\vec{a} \cdot \vec{b}) = m$
 Or $a b \cos \cos \theta = -m$
 Or $1 \cdot 1 \cdot \cos 120^\circ = -m$
 Therefore $m = 1/2$
9. Given $x^2 + 2y^2 + 3z^2 = 0 \therefore \phi = (x^2 + 2y^2 + 3z^2)$
 The unit normal is $\hat{n} = \frac{\vec{\nabla}\phi}{|\vec{\nabla}\phi|}$
 Where $\phi = x^2 + 2y^2 + 3z^2$
 $= 2x\hat{i} + 4y\hat{j} + 6z\hat{k}$
 $\therefore |\vec{\nabla}\phi| = \sqrt{4x^2 + 16y^2 + 36z^2}$
 $\hat{n} = \frac{\vec{\nabla}\phi}{|\vec{\nabla}\phi|_{(1,1,0)}} = \frac{2\hat{i} + 4\hat{j}}{\sqrt{20}} = \frac{\hat{i} + 2\hat{j}}{\sqrt{5}}$
10. Given that the two surfaces are orthogonal to each other.
 So their normal must also be orthogonal i.e. Perpendicular
 $\therefore \vec{\nabla}\phi = \vec{\nabla}(ax^2 + by^2 + z^2) = 2ax\hat{i} + 2by\hat{j} + 2z\hat{k}$
 $\vec{\nabla}\phi_2 = \vec{\nabla}(2ax^2 - by^2) = 40x\hat{i} - 2by\hat{j} + 0\hat{k}$
 $\vec{\nabla}\phi_1 \cdot \vec{\nabla}\phi_2|_{(1,1,1)} = 0$ or $8ax^2 - 4by^2|_{(1,1,1)} = 0$
 $\therefore b = \sqrt{2}a$
11. Here $\vec{P}\vec{\theta}$ is tangent to the surface and $\vec{\nabla}\phi$ is normal to the surface. $\therefore \vec{\nabla}\phi \cdot (\vec{P}\vec{\theta}) = 0$
 Where, $\vec{\nabla}\phi|_P = \vec{\nabla}(x^2 + y^2 + z^2)|_{1,2,3}$
 $= 2x\hat{i} + 2y\hat{j} + 2 = R$
 $= 2\hat{i} + 2y\hat{j} + 2 = R$
 & $\vec{P}\vec{\theta} = (x-1)\hat{i} + (y-2)\hat{j} + (z-1)\hat{k}$
 $\therefore \vec{\nabla}\phi \cdot \vec{P}\vec{\theta} = 0(2\hat{i} + 4\hat{j} + 2R) \cdot ((x+1)\hat{i} + (y-2)\hat{j} + (z-1)\hat{i}) = 0$
 or $x + 2y + z = 0$
12. $\vec{\nabla}\phi \cdot \vec{P}\vec{\theta} = 0$
 Where $\vec{P}\vec{\theta} = (x-1)\hat{i} + (y-1)\hat{j} + (z-1)\hat{k}$
 $\therefore \vec{\nabla}\phi \cdot \vec{P}\vec{\theta} = 0$ or $2x - 2 + y - 1 + z - 1 = 0$
 or $2x + y + z = 4$
13. Directional derivative of $\vec{A} = \vec{\nabla}\phi \cdot \vec{A}$
 $= (2x\hat{i} + 2y\hat{j} + 2z\hat{k})|_{(1,1,0)} \cdot \left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$
 $= \sqrt{3} + 1 + 0$
 $= \sqrt{3} + 1$
14. $\hat{A} = \cos \cos 60^\circ \hat{i} + \sin \sin 60^\circ \hat{j}$
 $= \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$
 $\therefore D.D.A = \vec{\nabla}\phi|_{(1,1)} \cdot \hat{A} = \left| \frac{x^2 + y^2 - x(2x)\hat{i} - x(2y)\hat{j}}{(x^2 + y^2)} \right|_{(1,1)} \cdot \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$
 $= \left(\frac{(1+1+2)\hat{i} - 2\hat{j}}{2}\right) \cdot \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$
 $= -\frac{\sqrt{3}}{2}$
15. Since, Directional derivative along $\hat{A} = \vec{\nabla}\phi|_P \hat{A}$
 $= |\vec{\nabla}\phi| |\hat{A}| \cos \cos \theta$
 $= |\vec{\nabla}\phi| \cos \cos \theta$
 $\therefore D.D_A = 0$ at $\theta = 90^\circ$ from the tangent of the surface.

16. Directional derivative of along $\hat{A} = \vec{\nabla}\phi \cdot \hat{A}$
 $= |\vec{\nabla}\phi| |\hat{A}| \cos \theta$
 $= |\vec{\nabla}\phi| \cos \theta$
 \therefore D.D will be maximum for $\theta = 0^\circ$
17. Since Divergence of $\phi = \vec{\nabla}\phi$ or $\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$
 So clearly divergence for a surface /scalar function tells us the rate of change of the scalar function normal to the surface
18. Since, Divergence of $\phi = \vec{\nabla}\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$
 If $\vec{\nabla}\phi = 0 \rightarrow$ not net outward flux \rightarrow solenoidal field
 If $\vec{\nabla}\phi =$ positive \rightarrow source field
 If $\vec{\nabla}\phi =$ negative \rightarrow sink field
19. $\vec{\nabla} \cdot \vec{r} \cdot \vec{r} = r^2 \cdot \vec{\nabla} \cdot \vec{r} + \vec{\nabla} \cdot \vec{r} \cdot \vec{r}$
 $= 3r^2 + 2r\hat{r} \cdot \vec{r}$
 $= 3r^2 + 2r^2 = 5r^2$
20. Using: $\vec{\nabla} \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \vec{\nabla})\vec{A} - (\vec{A} \cdot \vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla} \cdot \vec{B}) - \vec{B}(\vec{\nabla} \cdot \vec{A})$
 \therefore For constant vector field \vec{a}
 $\vec{\nabla} \times \vec{a} \times \hat{r} = (\hat{r} \cdot \vec{\nabla})\vec{a} - (\vec{a} \cdot \vec{\nabla})\hat{r} + \vec{a}(\vec{\nabla} \cdot \hat{r}) - \hat{r}(\vec{\nabla} \cdot \vec{a})$
 $= 0 - \vec{a} + 3\vec{a} - 0$
 $= 2\vec{a}$
21. $\vec{\nabla}(\vec{a} \cdot \vec{r}) = \vec{\nabla}(a_x x + a_y y + a_z z)$
 $\frac{\partial}{\partial x}(a_x x)\hat{i} + \frac{\partial}{\partial y}(a_y y)\hat{j} + \frac{\partial}{\partial z}(a_z z)\hat{k}$
 $= a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$
 $= \vec{a}$
22. Since we know that the work done by a continuous force field along a closed path is zero, whatever be the form of the closed curve.
23. Since we know that the work done by continuous force field along a closed path is zero, whatever be the form of the closed curve. But it is valid only when, within the closed curve, the vector field is defined at all points
 Here \vec{A} is not defined only at origin which is not within the curve region
24. The given vector field is conservation but since within the curve, it is not defined at the origin S, we have the work done won't be zero
 So, we have to calculate:
 $\oint_0 \vec{F} \cdot d\vec{r} = \oint_c \frac{-ydx + xdy}{x^2 + y^2}$
 $= \int_0^{2\pi} \frac{-R \sin \theta (-R \sin \theta) d\theta + R \cos \theta R \cos \theta d\theta}{R^2}$
 $= \int_0^{2\pi} d\theta = 2\pi$

25. Since, $\vec{\nabla} \cdot \vec{A} = -2x + 1$ which is non zero

$$\text{and } \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x^2 & -y & 0 \end{vmatrix} = 0$$

- $\therefore \vec{A} = -(x^2\hat{i} + y\hat{j})$ is irrotational.

26. Condition of \vec{A} being solenoidal field is $\vec{\nabla} \cdot \vec{A} = 0$ or
 $-m + 1 = 0$

- $\therefore m = 1$ is the required value of m

27. For a vector field to be irrotational

$$\vec{\nabla} \times \vec{A} = 0 \text{ or } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -my & x & 0 \end{vmatrix} = 0 \text{ or } \hat{i}(0) - \hat{j}(0) +$$

$$\hat{k}(1 + m) = 0$$

$$\text{Or } m = -1$$

28. $\oint_c \vec{F} \cdot d\vec{r} = \iint_s (\vec{\nabla} \times \vec{F}) \cdot d\vec{s}$ Using stroke's theorem

$$\text{Where } \vec{\nabla} \times \vec{F} = \vec{\nabla} \times \vec{r} \times \vec{a}$$

$$= -\vec{\nabla} \times \vec{a} \times \vec{r} \quad (\because \vec{\nabla} \times \vec{a} \times \vec{r} = 2\vec{a} \text{ Where } \vec{a} = \text{constant vector})$$

$$\int_s (\vec{\nabla} \times \vec{F}) \cdot d\vec{s} = -2a_n \int \int ds$$

$$= -2a_n \pi (1)^2$$

$$= -2a_n \pi$$

29. For C1: $x = y \therefore dx = dy$

$$\therefore \oint_{c_1} \vec{F} \cdot d\vec{r} = \int_{x=0}^{4a} (x^2 + x^2) dx + x^2 dx$$

$$= -64a^3$$

$$\text{For C2: } x = \frac{y^2}{4a} \therefore dx = \frac{y}{2a} dy$$

$$\int_{c_2} \vec{F} \cdot d\vec{r} = \int_{y=0}^{4x} (y \cdot \frac{y^2}{4a} + y^2) \frac{y}{2a} dy + \frac{y^n}{16a^2} dy$$

$$= \int_{y=0}^{4a} \left[\frac{y^4}{8a^2} + \frac{y^4}{16a^2} + \frac{y^3}{2a} \right] dy$$

$$= \frac{3}{16a^2} \left(\frac{y^5}{5} \right) + \frac{1}{2a} \frac{y^4}{4} \Big|_0^{4a} = 38.4a^3 + 32a^3$$

$$= 70.4 a^3$$

$$\oint \vec{F} \cdot d\vec{r} = \int_{c_1} \vec{F} \cdot d\vec{r} + \int_{c_2} \vec{F} \cdot d\vec{r} = 70.4a^3 + (-64a^3)$$

$$= 6.4 a^3$$

30. Flux = $\oint_s \vec{F} \cdot d\vec{s} = \iiint_v (\vec{\nabla} \cdot \vec{F}) dV$

$$= \iiint_{0,0,0}^{2,2\pi,3} (4 - 4y + 2z) dV$$

$$= \iiint_{0,0,0}^{2,2\pi,3} (4 - 4y + 2z) s ds d\phi dz$$

$$= \iiint_{0,0,0}^{2,2\pi,3} (4 - 4s \sin \phi + 2z) s ds d\phi dz$$

$$\text{Where the limit will be } \therefore s: 0 \rightarrow 2, \phi: 0 \rightarrow 2\pi, z: 0 \rightarrow 3$$

$$= \int_{s=0}^2 \int_{\phi=0}^{2\pi} \int_{z=0}^3 (4 - 4 \sin \phi + 2z) s ds d\phi dz$$

$$= 84 \pi$$

Unit 1.3: Matrices

1.3.1: Matrices

- What are the only possible eigenvalues of a 2×2 idempotent matrix?
 - 0 and 3
 - 1 and 2
 - 0 and 1
 - none of these
- For a $n \times n$ nilpotent matrix M , one of the eigenvalues have to be
 - 1
 - 0
 - n
 - none of these
- Which of the following set of vectors are linearly dependent
 - $\begin{bmatrix} 0 \\ -5i \\ -i \end{bmatrix}, \begin{bmatrix} 5i \\ 0 \\ -2i \end{bmatrix}$ and $\begin{bmatrix} i \\ 2i \\ 0 \end{bmatrix}$
 - $\begin{bmatrix} 0 \\ 5i \\ -i \end{bmatrix}, \begin{bmatrix} i \\ 1 \\ -2i \end{bmatrix}$ and $\begin{bmatrix} 5i \\ 2i \\ i \end{bmatrix}$
 - $\begin{bmatrix} 0 \\ 5i \\ -i \end{bmatrix}, \begin{bmatrix} 5i \\ 0 \\ -2i \end{bmatrix}$ and $\begin{bmatrix} i \\ i \\ 0 \end{bmatrix}$
 - $\begin{bmatrix} 2i \\ -5i \\ -i \end{bmatrix}, \begin{bmatrix} i \\ 0 \\ -2i \end{bmatrix}$ and $\begin{bmatrix} i \\ 2i \\ 0 \end{bmatrix}$
- Consider A and B be the symmetric matrix such that commutator of A and B is 0, then $[AB]^T$ will be
 - AB
 - BA
 - $[AB]^{-1}$
 - $[BB]^{-1}$
- For a 3×3 symmetric matrix M , the number of independent components will be
 - 4
 - 3
 - 6
 - 9
- For a 3×3 skew symmetric matrix M , the number of independent components will be
 - 4
 - 3
 - 6
 - 9
- For $k = a + ib$, where a and b are non-zero constants. What will be the nature of (kM) , considering M being a Hermitian matrix
 - Hermitian
 - Skew Hermitian
 - None of these
 - $\text{Det} = 0$
- Given that a matrix M has its determinant 3, then what will be the value of determinant of inverse of M
 - $\frac{1}{3}$
 - 3
 - $\frac{1}{9}$
 - 9
- Consider a Matrix M , such that all of its elements are given by $a_{ij} = -ij$, Given that $M = \begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix}$
 - 0, -1
 - 1, -5
 - 5, 0
 - 0, -5
- For a matrix, the necessary condition is that each row and each column is normalized, the nature of matrix will be
 - Orthogonal
 - Skew Hermitian
 - Hermitian
 - Idempotent
- The eigenvalues of the matrix $M = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ at $\theta = 30^\circ$
 - $\frac{\sqrt{3}}{2} \pm \frac{i}{2}$
 - $\frac{i}{2} \pm \frac{\sqrt{3}}{2}$
 - $\sqrt{2} + i$
 - $i \pm \sqrt{2}$
- For a Matrix $M, (i(M - M^T))^T$ is
 - Orthogonal
 - Hermitian
 - Identical
 - Idempotent
- What will be the commutator of two matrices

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
 - BC
 - CB
 - B^{-1}
 - 0
- What will be the commutator of two matrices

$$B = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
 - $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
 - $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 - $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
 - $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- Consider A to be a 3×3 Hermitian matrix having trace=5, then number of independent parameters of A is
 - 7
 - 5
 - 8
 - 9
- The characteristic equation of a 3×3 matrix is $A^3 - 3A^2 + 3A + 7I = 0$. Then $\text{Det}(A^{-1})$ will be
 - 2
 - 7
 - 2
 - 7