P R E F A C E

Welcome to the comprehensive compilation of JEST (Joint Entrance Screening Test) Physics previous year solutions. This book has been meticulously crafted to serve as an indispensable tool for aspiring physicists preparing for the prestigious JEST examination.

The Joint Entrance Screening Test (JEST) is a national level entrance examination conducted annually for admissions into Ph.D. and integrated Ph.D. programs in Physics, Theoretical Computer Science in premier research institutes across India. Aspiring candidates aiming to embark on a journey in the field of physics undergo rigorous preparation to excel in this competitive examination.

This book stands as a testament to the dedication and hard work put forth by countless aspirants in their quest to master the intricacies of physics. It is designed to provide a comprehensive solution set to the previous year's questions, thereby aiding students in understanding the pattern, structure, and nuances of the JEST examination.

One of the unique features of this book is the meticulous organization of solutions into subtopics, facilitating a structured approach towards learning. Each subtopic is analysed thoroughly, providing insights into the types of questions frequently asked, their level of difficulty, and the underlying concepts they assess. This subtopic-wise sorting and analysis serve as invaluable guidance for students, enabling them to focus their preparation efficiently and effectively.

Furthermore, this book doesn't merely offer solutions; it endeavours to cultivate a deeper understanding of physics concepts. Through detailed explanations, alternate problem-solving approaches, and insightful tips, students are encouraged to delve into the underlying principles, fostering a holistic grasp of the subject matter.

It's imperative to acknowledge the collaborative effort that has gone into the creation of this resource. The dedication of educators, researchers, and students who have contributed to compiling and reviewing the solutions is commendable. Their expertise and commitment have been instrumental in ensuring the accuracy and quality of this book.

As you embark on your journey through the solutions presented herein, may you find clarity, inspiration, and confidence in your pursuit of excellence. Let this book be your guiding companion, empowering you to conquer the challenges of the JEST examination and unlock the doors to a promising future in the realm of physics.

Best wishes,

IFAS Publication

A C K N O W L E D G E M E N T

I would like to express my heartfelt gratitude to **IFAS Edu.pvt.Ltd, IFAS publications, Dr. Kailash Choudhary, Er. Radheshyam Choudhary & Mr. Dadasaheb Sondge** who have contributed to the successful publication of my book on the solutions to the previous year's JEST (Joint Entrance Screening Test) questions. Your support, encouragement, and assistance have been invaluable throughout this journey.

First and foremost, I extend my deepest appreciation to my Subject Matter Team: **Miss. Ankita Sable, Mr. Kishor Kalarakoppa, and Mr. Yogesh Malviya** whose unwavering guidance and expertise played a pivotal role in shaping this publication. Your insights and feedback have significantly enhanced the quality of the content, making it more comprehensive and accessible to readers.

I am immensely grateful to **IFAS Publications pvt.Ltd**. For providing the resources and platform necessary for the development and publication of this book. Your belief in my work has been a constant source of motivation.

I extend my sincere thanks to all the contributors, reviewers, and experts who generously shared their knowledge and expertise to ensure the accuracy and reliability of the solutions presented in this book. Your dedication to academic excellence has been instrumental in making this project a success.

Last but not least, I would like to express my gratitude to the readers and students who will benefit from this book. It is my sincere hope that the solutions provided herein will serve as a valuable resource in your preparation for the JEST examination, and contribute to your academic success.

Thank you once again to everyone who has contributed to the realization of this project. Your support and encouragement have made all the difference, and I am deeply grateful for the opportunity to share this work with the academic community.

With warm regards,

POOJA PATIL

Joint Entrance Screening Test
Syllabus

Mathematical Methods

Vector algebra and vector calculus, tensors, curvilinear coordinate systems, linear algebra;
Linear differential equations, elements of Sturm–Liouville theory;
Special functions; Complex analysis; Fourier series and Fourier transforms, Laplace transforms;
Elementary properties of discrete groups; Elements of probability theory, error analysis.

Classical Mechanics

Newton's laws, conservation of energy and momentum, collisions;
generalized coordinates, principle of least action,
Lagrangian and Hamiltonian formulations of mechanics;
Symmetry and conservation laws; central force problem, Kepler problem;
Small oscillations and normal modes; special relativity in classical mechanics.

Electromagnetism & Optics

Electrostatics and magnetostatics, boundary value problems, multipole expansion;
Fields in conducting, dielectric, diamagnetic and paramagnetic media;
Faraday's law and time varying fields; displacement current;
Maxwell's equations; energy and momentum of electromagnetic fields;
Propagation of plane electromagnetic waves, reflection, refraction;
Electromagnetic waves in dispersive and conducting media;
diffraction, interference, polarization.

Quantum Mechanics

Uncertainty principle; Schrodinger equation; central potentials, hydrogen atom;
Orbital and spin angular momenta, addition of angular momenta;
Matrix formulation of quantum theory, unitary transformations, Hermitian operators;
Variational principle, time independent perturbation theory, time dependent perturbation theory.

Thermodynamics & Statistical Physics

Laws of thermodynamics, work and heat, thermodynamic potentials;
Elements of kinetic theory; Maxwell’s relations;
Statistical ensembles; partition function; classical ideal gas, harmonic oscillators;
Classical and quantum statistics; Fermi and Bose gases;
black body radiation; statistics of paramagnetism

Electronics

Basics of semiconductor; p-n junctions, diodes, transistors;
LCR circuits, rectifiers, amplifiers, active filters and oscillators;
basics of OPAMPs and their applications; basics of digital electronics.

YEAR WISE CUT-OFF MARKS:

YEAR	COURSE	CUT OFF MARKS
2024	I-PhD	30.33
	PhD	36
2022	I-PhD	21.67
	PhD	26.67
2021	I-PhD	19.36
	PhD	23.03
2020	I-PhD	26
	PhD	32
2019	I-PhD	35
	PhD	32

Paper pattern of JEST 2024:

Section	Part A(MCQ)	Part B (MCQ)	Part C (NAT)
No. of Questions	25	15	10
Marks per question	1	3	3
Negative marking	-1/3	-1	NO

Paper pattern of JEST 2023:

Section	Part A(MCQ)	Part B (MCQ)
No. of Questions	25	25
Marks per question	3	9
Negative marking	-1	-3

Paper pattern of JEST 2022:

Section	Part A(MCQ)	Part B (MCQ)	Part C (NAT)
No. of Questions	25	15	10
Marks per question	1	3	3
Negative marking	-1/3	-1	NO

Paper pattern of JEST 2021:

Section	Part A(MCQ)	Part B (MCQ)	Part C (NAT)
No. of Questions	25	15	10
Marks per question	1	3	3
Negative marking	-1/3	-1	NO

Paper pattern of JEST 2020:

Section	Part A(MCQ)	Part B (MCQ)	Part C (NAT)
No. of Questions	25	15	10
Marks per question	1	3	3
Negative marking	-1/3	-1	NO

Paper pattern of JEST 2019:

Section	Part A(MCQ)	Part B (MCQ)	Part C (NAT)
No. of Questions	25	15	10
Marks per question	1	3	3
Negative marking	-1/3	-1	NO

Eligibility

- **Please see the websites/advertisements of the participating institutes for their eligibility criteria in details.**
- Listed below are tentative eligibility criteria of admission to M.Sc, Ph.D, and Integrated / M.Sc. / M.Tech. - Ph.D programs in the participating institutes.

Ph.D. Programme

Physics

- M.Sc. in Physics (**all participating Institutes**).

Additionally, some institutes accept B.E. / B.Tech. / M.Sc. / M.E. / M.Tech. In disciplines of Applied Physics and Mathematics, as listed below.

- M.Sc. in Mathematics / Applied Physics / Applied Mathematics / Optics and Photonics / Instrumentation / Electronics will also be considered **at IIA**.
- B.E. or B.Tech. Will be considered at **IISc, IMSc, ICTS-TIFR, IUCAA, JNCASR, NCRA-TIFR, TIFR-TCIS, RRI, IISER Mohali, IISER Pune, and IISER Thiruvananthapuram**.
- M.Sc. in Physics / Electronics / Astronomy / Applied Mathematics will be considered at **IUCAA and NCRA-TIFR.**
- MSc in Physics, Engineering Physics or Applied Physics will also be considered at **IPR.**
- M.Sc. in Physics, Chemistry, Applied Mathematics, Biophysics or Biochemistry will be considered at **SNBNCBS**.
- B.Tech Eng. Phys.will be considered at **TIFR and NCRA-TIFR.**..
- M.E. / M.Tech in Applied Physics will be considered at **NISER**.
- M.Sc. in Physics/Astronomy/Optics/Photonics/Biophysics/Mathematics or ME/M.Tech in Engineering also will be considered at **RRI**.
- MSc. in Physics, Chemistry and Bio Physics will be considered at **SNBNCBS**.

Theoretical Computer Science at IMSc

- M.Sc. / M.E. / M.Tech. In Computer Science and related disciplines, and should be interested in the mathematical aspects of computer science.

Ph.D in Neuroscience at NBRC

- M.Sc (Physics/ Mathematics), B.E/ B.Tech/ M.C.A in Computer Science

Ph.D in Computational Biology at IMSc

- M.Sc. / M.E. / M.Tech. / MCA in any engineering or science discipline, with good mathematical skills and strong interest in biological problems.

Integrated M.Sc. / M.Tech - Ph.D Programme (Physics)

- BSc in Physics (Major/Hons), or in Physics and Mathematics (Physics cannot be an allied/subsidiary/minor subject) will be considered at **SNBNCBS**.
- B. Sc. (Physics) will be considered at **IMSc**.
- B.Sc. (Physics / Mathematics) / B.E. / B.Tech. In Electrical / Instrumentation / Engineering Physics / Electronics and Communications / Computer Science and Engineering / Optics and Photonics will be considered in **IIA**.

- B.Sc. (Physics) or B.E. /B. Tech in Engineering Physics, with a minimum of first class marks, will be considered at **NISER**.
- B.Sc. (Physics) will be considered at **IISER-Pune, ICTS-TIFR, NCRA-TIFR**, and **TIFR-TCIS**.
- B. Sc. (Physics / Mathematics) / B.E. / B.Tech. Will be considered for Integrated M.Sc - PhD at **Bose Institute**.
- B. Sc. (Physics / Mathematics/Computer Science/Electronics) / B.E. / B.Tech. Will be considered for Integrated M.Sc - PhD at **Bose Institute**.

Integrated Ph.D Programme in Theoretical Computer Science at IMSc

- B.Sc. /B.E. /B.Tech. /M.C.A. in Computer Science or related disciplines and should be interested in the mathematical aspects of computer science.

Integrated M.Tech - Ph.D. Programme at IIA

- M.Sc. (Physics / Applied Physics) / Post-B.Sc. (Hons) in Optics and Optoelectronics / Radio Physics and Electronics.

Integrated Ph.D. Programme at IISER Kolkata and IISER. Thiruvananthapuram.

- B. Sc. (Physics) or B.E. / B. Tech. in any discipline.

M. Sc. Programme at HRI.

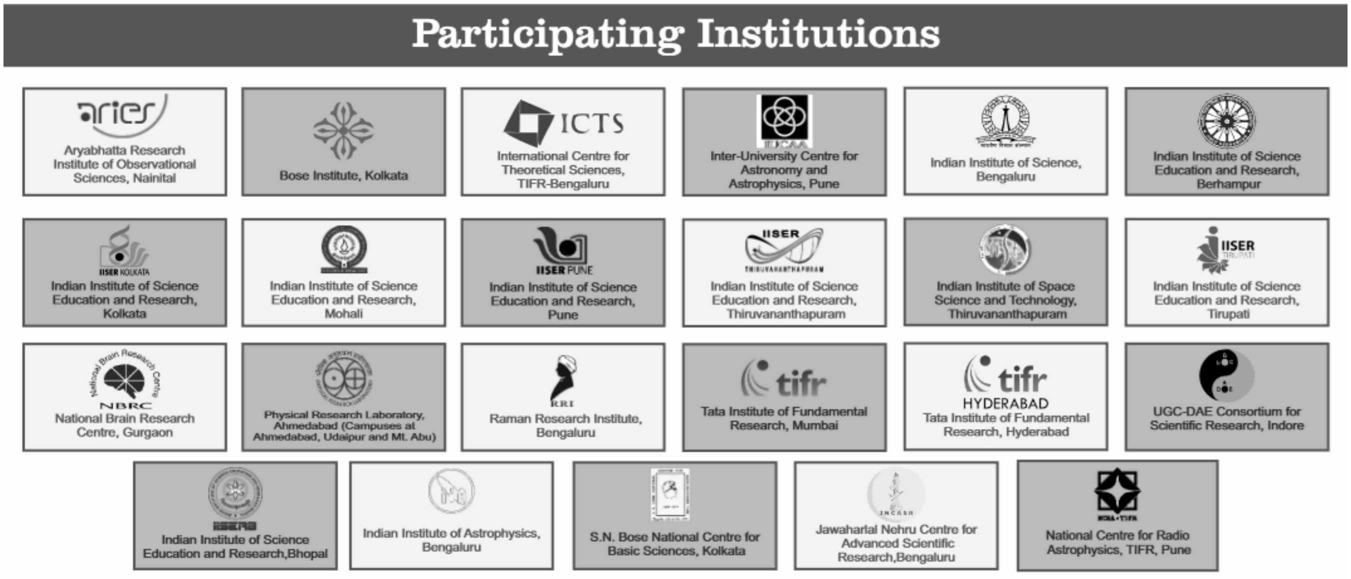
- B. Sc. (Physics) or B.E./B.Tech. degree in any discipline
- HRI is starting a new M.Sc. programme in Physics from 2017. The Integrated Ph.D. programme in Physics at HRI is discontinued from 2017.

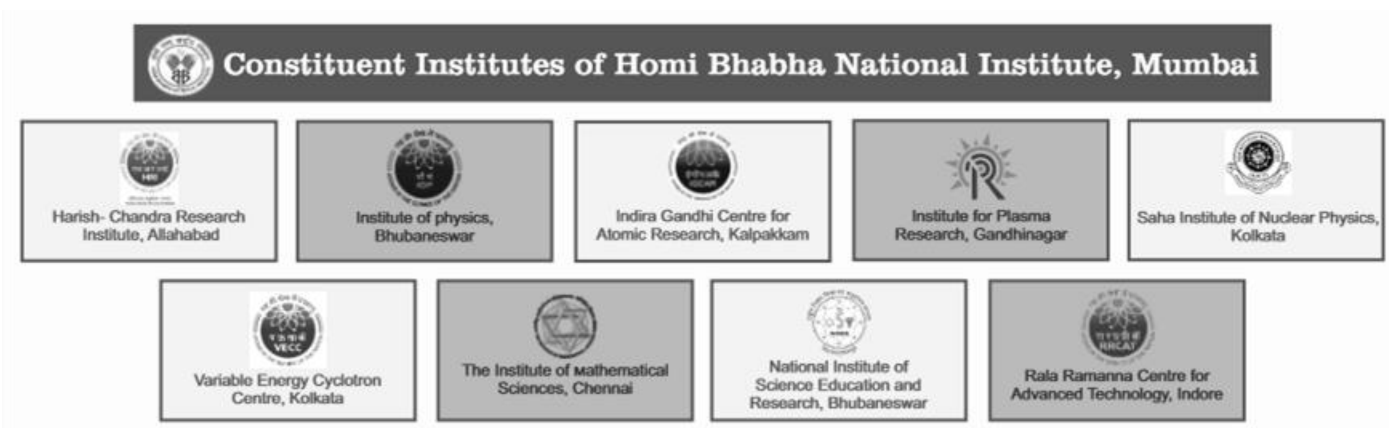
Joint SPPU-IUCAA MSc Physics (Astrophysics) programme

- B. Sc. (Physics and Mathematics) or B.E. / B. Tech. in any branch of engineering, with a minimum of 55% marks.

Participating Institutes

Visit the individual institute pages to view the programmes and subject areas being offered.





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JEST PHYSICS

MATHEMATICAL METHODS IN PHYSICS

No. of questions asked from year 2012-2024

Subtopics		2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
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2	VECTOR ALGEBRA & VECTOR CALCULUS	-	1	-	-	-	1	1	2	-	2	1	3	-
3	MATRICES	1	-	-	-	1	2	1	3	1	2	1	2	2
4	LINEAR ORDINARY DIFFERENTIAL EQUATIONS OF 1st AND 2nd ORDER	1	-	1	3	1	1	2	-	2	1	1	1	1
5	SPECIAL FUNCTIONS	-	1	-	1	-	-	1	2	-	-	-	-	-
6	FOURIER SERIES, FOURIER TRANSFORM & LAPLACE TRANSFORM	2	1	2	-	1	2	1	-	1	2	-	1	1
7	COMPLEX ANALYSIS	1	1	-	1	-	2	1	3	2	-	2	2	1
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MATHEMATICAL METHODS IN PHYSICS

CHAPTER 1

JEST PHYSICS

(PREVIOUS YEAR EXAM QUESTIONS)

DIMENSIONAL ANALYSIS

[2014]

1. Let us write down the Lagrangian of a system as $L(x,\ddot{x},\dddot{x}) = mx\ddot{x} + kx^2 + cx\dddot{x}$. What is the dimension of c ?

(a) MLT^{-3} (b) MT^{-2}

(c) MT (d) ML^2T^{-1}

[2014]

2. Given the fundamental constants $\hbar$ (Planck's constant), G (universal gravitation constant) and c (speed of light), which of the following has dimension of length?

(a) $\sqrt{\frac{\hbar G}{c^3}}$ (b) $\sqrt{\frac{\hbar G}{c^5}}$

(c) $\frac{\hbar G}{c^3}$ (d) $\sqrt{\frac{\hbar c}{8\pi G}}$

[2014]

3. The Dirac delta function $\delta(x)$ satisfies the relation $\int_{-\infty}^{\infty} f(x)\delta(x)\,dx = f(0)$ for a well-behaved function $f(x)$. If x has the dimension of momentum then

(a) $\delta(x)$ has the dimension of momentum

(b) $\delta(x)$ has the dimension of (momentum)^2

(c) $\delta(x)$ is dimensionless

(d) $\delta(x)$ has the dimension of (momentum)$^{-1}$

Answer Key		
1	2	3
(c)	(a)	(d)

SOLUTIONS: DIMENSIONAL ANALYSIS

1. Solution: According to dimension rule same dimension will be added or subtracted then dimension of

$\mathrm{Mx\ddot{x}} = \text{dimension of } \mathrm{Cx\dddot{x}}$

$[\mathrm{ML^2T^{-2}}] = [\mathrm{C}][\mathrm{L}][\mathrm{LT^{-3}}]$

$[\mathrm{C}] = \frac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{L^2T^{-3}}]} = [\mathrm{MT}]$

Hence, option (c) is correct.

2. Solution: we check the options first, then we get

$\left[\frac{[ML^2T^{-1}][M^{-1}L^3T^{-2}]}{L^3T^{-3}}\right]^{1/2} = [L^2]^{\frac{1}{2}} = L$

$h = [ML^2T^{-1}]$, $G = \frac{gr^2}{m} = [M^{-1}L^3T^{-2}]$

Hence, option (a) is correct.

3. Solution: $\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$

$f(x)\delta(x)dx = f(0) \Rightarrow [f(x)]\delta(x).P = [f(0)] \Rightarrow$

$\delta(x) = [P^{-1}]$

$since\,, [f(x)] = [f(0)]$

$If\ F(x) = \alpha x + \beta\ is\ force\ [MLT^{-2}]$

$F(0) = \beta\ \ is\ also\ [MLT^{-2}]$

Hence, option (d) is correct.

VECTOR ALGEBRA & VECTOR CALCULUS

[2013]

1. The vector field $xz\hat{\imath} + y\hat{\jmath}$ in cylindrical polar coordinates is

(a) $\mathrm{p}(\mathrm{z}\cos^2\phi + \sin^2\phi)\hat{\mathrm{e}}_\mathrm{p} + \mathrm{p}\sin\phi\cos\phi\,(1-\mathrm{z})\hat{\mathrm{e}}_\phi$

(b) $\mathrm{p}(\mathrm{z}\cos^2\phi + \sin^2\phi)\hat{\mathrm{e}}_\mathrm{p} + \mathrm{p}\sin\phi\cos\phi\,(1+\mathrm{z})\hat{\mathrm{e}}_\phi$

(c) $\mathrm{p}(\mathrm{z}\cos^2\phi + \sin^2\phi)\hat{\mathrm{e}}_\mathrm{p} + \mathrm{p}\sin\phi\cos\phi\,(1-\mathrm{z})\hat{\mathrm{e}}_\phi$

(d) $\mathrm{p}(\mathrm{z}\sin^2\phi + \cos^2\phi)\hat{\mathrm{e}}_\mathrm{p} + \mathrm{p}\sin\phi\cos\phi\,(1-\mathrm{z})\hat{\mathrm{e}}_\phi$

[2017]

2. What is the equation of the plane which is tangent to the surface xyz = 4 at the point (1, 2, 2)?

(a) $x + 2y + 4z = 12$ (b) $4x + 2y + z = 12$

(c) $x + 4y + 9z = 0$ (d) $2x + y + z = 6$

[2018]

3. $\pi\int_{-\infty}^{\infty} \exp(-|x|)\delta\ (\sin(\pi x))\,dx$, where $\delta(\dots)$ is Dirac distribution, is

(a) 1 (b) $\frac{e+1}{e-1}$

(c) $\frac{e-1}{e+1}$ (d) $\frac{e}{e+1}$

[2019]

4. Suppose $\psi\vec{A}$ is a conservative vector, $\vec{A}$ is a non-conservative vector and ψ is non-zero scalar everywhere. Where one of the following is true?

(a) $(\nabla \times \hat{A})\,.\,\hat{A} = 0$ (b) $\vec{A} \times \nabla\psi = \vec{0}$

(c) $\vec{A}.\nabla\psi = 0$ (d) $(\nabla \times \vec{A}) \times \vec{A} = \vec{0}$

[2019]

5. What is the angle (in degrees) between the surfaces $y^2 + z^2 = -2$and $y^2 - x^2 = 0$at the point (1, -1, 1)

[2021]

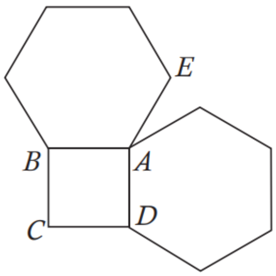
6. Let ABCDEF be a regular hexagon. The vector $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$ will be

(a) 0 (b) $\overrightarrow{AD}$

(c) $2\overrightarrow{AD}$ (d) $3\overrightarrow{AD}$

[2021]

7. A paper has been cut into the shape given in figure (ABCD is a square and the two hexagonal flaps are regular) and placed on the table. The square base lies flat on the table. The hexagonal flaps are then folded upwards along the edges AB and AD such that edges AE and AF of the two hexagons coincide. What is the minimum angle (in degrees) made by the edge AE (or AF) with the surface of the table?



(a) 120 (b) 85
(c) 60 (d) 45

[2022]

8. If θ and ϕ are respectively the polar and azimuthal angles on the unit sphere, what is $\langle\cos^2(\theta)\rangle$ and $\langle\sin^2(\theta)\rangle$, where $\langle O\rangle$ denotes the average of O?
(a) $\langle\cos^2(\theta)\rangle = \frac{2}{3}$ and $\langle\sin^2(\theta)\rangle = \frac{1}{3}$
(b) $\langle\cos^2(\theta)\rangle = \frac{1}{2}$ and $\langle\sin^2(\theta)\rangle = \frac{1}{2}$
(c) $\langle\cos^2(\theta)\rangle = \frac{3}{4}$ and $\langle\sin^2(\theta)\rangle = \frac{1}{4}$
(d) $\langle\cos^2(\theta)\rangle = \frac{1}{3}$ and $\langle\sin^2(\theta)\rangle = \frac{2}{3}$

[2023]

9. The position and velocity vectors of a particle changes from $\vec{R_1}$ to $\vec{R_2}$ and $\vec{v_1}$ to $\vec{v_1}$ respectively as time flows from t_1 to t_2. If $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ are the instantaneous position, velocity and acceleration vectors of the particle, compute the integral: $\vec{I} = \int_{t_1}^{t_2} \vec{r} \times \vec{a}\, dt$. Mark the correct answer.
(a) $\vec{I} = \vec{R_2} \times \vec{v_2} - \vec{R_1} \times \vec{v_1}$
(b) $\vec{I} = \vec{R_1} \times \vec{v_1} - \vec{R_1} \times \vec{v_2}$
(c) $\vec{I} = 0$
(d) $|\vec{I}| = |\vec{R_1} \times \vec{v_1}| + |\vec{R_2} \times \vec{v_2}|$

[2023]

10. Which of the following vanishes identically?
(a) $\nabla \times \frac{((y+x)\hat{\imath}+(y-x)\hat{\jmath})}{x^2+y^2}$ (b) $\nabla \times \frac{(y\hat{\imath}-x\hat{\jmath})}{x^2+y^2}$
(c) $\nabla \times \frac{(x\hat{\imath}+y\hat{\jmath})}{x^2+y^2}$ (d) $\nabla \cdot \left[\frac{(x\hat{\imath}+y\hat{\jmath}+z\hat{k})}{(x^2+y^2+z^2)^{3/2}}\right]$

[2023]

11. Given the vector $\vec{v} = y\hat{\imath} + 3x\hat{\jmath}$, what is the value of the integral $\oint \vec{v}\cdot d\vec{r}$ along the unit circle (centered at the origin) in an anti-clockwise direction?
(a) $\frac{2\pi}{3}$ (b) π
(c) 0 (d) 2π

Answer Key									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
(a)	(d)	(b)	(a)	60	(d)	(d)	(d)	(a)	(c)
11									
(d)									

SOLUTION: VECTOR ALGEBRA & VECTOR CALCULUS

1. **Solution:** $\vec{A} = xz\hat{i} + y\hat{j} \Rightarrow A_x = xz,\ A_y = y,\ A_z = 0$
$A_\rho = \vec{A}.\hat{e}_\rho = A_x(\hat{x}.\hat{e}_\rho) + A_y(\hat{y}.\hat{e}_\rho) + A_z(\hat{z}.\hat{e}_\rho)$
$A_\rho = \rho\cos\phi z(\cos\phi) + \rho\sin\phi(\sin\phi) + 0 \Rightarrow A_\rho = (\rho\cos\phi^2 z + \rho\sin^2\phi)\hat{e}_\rho$
$A_\phi = \vec{A}.\hat{e}_\phi = A_x(\hat{x}.\hat{e}_\phi) + A_y(\hat{y}.\hat{e}_\phi) + A_z(\hat{z}.\hat{e}_\phi)$
$A_\phi = \rho\cos\phi(-\sin\phi)z + \rho\sin\phi.\cos\phi \Rightarrow$
$A_\phi \rho\cos\phi.\sin\phi(1-z)\,\hat{e}_\phi$
$\vec{A} = A_\rho\hat{e}_\rho + A_\phi\hat{e}_\phi + A_z\hat{e}_z = \rho(z\cos^2\phi + \sin^2\phi)\hat{e}_\rho + \rho\cos\phi\sin\phi(1-z)\hat{e}_\phi$
Hence, option (a) is correct.

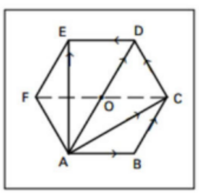
2. **Solution:** The surface equation is given by
$\phi = xyz = 4$
The normal vector to the surface is
$\vec{n} = \vec{\nabla}\phi = yz\hat{x} + xz\hat{y} + xy\hat{z}$
At point (1, 2, 2),
$\vec{n} = (4\hat{x} + 2\hat{y} + 2\hat{z}) \Rightarrow \hat{n} = \frac{\vec{n}}{|n|} = \frac{(4\hat{x}+2\hat{y}+2\hat{z})}{\sqrt{16+4+4}} = \frac{(2\hat{x}+\hat{y}+\hat{z})}{\sqrt{6}}$
The equation of plane at point (1, 2, 2), is
$[(x-1)\hat{x} + (y-2)\hat{y} + (z-2)\hat{z}]\hat{n} = 0$
$\Rightarrow 2(x-1) + (y-2) + (z-2) = 0 \Rightarrow 2x + y + z = 6$
Hence, option (d) is correct.

3. **Solution:** We know that $\delta[f(x)] = \sum_i \frac{\delta(x-x_i)}{|f'(x_i)|}$,
Where x_i's are the root of the equation $f(x) = 0$
Therefore, $sin\pi x = 0 \Rightarrow \pi x = nx \Rightarrow x_i = n$ where n is an integer
$f'(x) = \pi cos\pi x \Rightarrow |f'(x_i)| = |\pi(-1)^n| = \pi$
$Hence\ , \pi\int_{-\infty}^{\infty} \exp(-|x|)\,\delta\sin(\pi x)\,dx$
$= \int_{-\infty}^{\infty} \exp(-|x|)\,[\delta(0) + \delta(x-1) + \delta(x+1)\delta(x-2) + \delta(x+2) + \cdots]$
$= e^{-|0|} + e^{-|1|} + e^{-|-1|} + e^{-|2|} + e^{-|-2|} + \cdots = 1 + e^{-1} + e^{-1} + e^{-2} + e^{-2} + \cdots$
$1 + 2(e^{-1} + e^{-2} + e^{-3} + \cdots)$
The terms in the bracket form geometric series with first term e^{-1} $and\ common\ ratio\ e^{-1}$.
$\pi\int_{-\infty}^{\infty} \exp(-|-x|)\,\delta\sin(\pi x)\,dx = 1 + 2\pi \quad \frac{e^{-1}}{1-e^{-1}} = 1 + 2\frac{\frac{1}{e}}{1-\frac{1}{e}}$
$= 1 + 2\frac{\frac{1}{e}}{\frac{(e-1)}{e}} = 1 + 2.\frac{1}{e-1} = \frac{e-1+2}{e-1} = \frac{e+1}{e-1}$
Hence, option (b) is correct.

4. **Solution:** As we know that divergence of a curl is always ZERO.
Hence, option (a) is correct.

5. **Solution:** The equation of two surface is
$f(x,y,z) = 2$ and $g(x,y,z) = 0$
Where $f(x,y,z) = y^2 + z^2$ and $g(x,y,z) = y^2 = x^2$
The normal to the first surfaces is
$$\overrightarrow{\nabla f} = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k} \Rightarrow \overrightarrow{\nabla f} = 2y\hat{\jmath} + 2z\hat{k}$$
$$\overrightarrow{\nabla g} = \frac{\partial g}{\partial x}\hat{\imath} + \frac{\partial g}{\partial y}\hat{\jmath} + \frac{\partial g}{\partial z}\hat{k} \Rightarrow \overrightarrow{\nabla g} = -2x\hat{\imath} + 2y\hat{\jmath}$$
At point $(1,-1,1), \overrightarrow{\nabla g} = -2\hat{\jmath} + 2\hat{k}$ *and* $\overrightarrow{\nabla g} = -2\hat{\imath} - \hat{2\jmath}$
Hence the angle between the two surface is
$$\theta = \cos^{-1}\frac{\overrightarrow{\nabla f}.\overrightarrow{\nabla g}}{|\overrightarrow{\nabla f}||\overrightarrow{\nabla g}|} = \cos^{-1}\frac{(-2\hat{\jmath} + 2\hat{k}).(-2\hat{\imath} - 2\hat{\jmath})}{\sqrt{8}\sqrt{8}}$$
or $\theta = \cos^{-1}\frac{4}{8} = \cos^{-1/2} = 60^0$
Hence, answer is 60

6. **Solution:** $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$
$= (\overrightarrow{AC} + \overrightarrow{AF}) + \overrightarrow{AD} + (\overrightarrow{AE} + \overrightarrow{AB})$
$= (\overrightarrow{AC} + \overrightarrow{CD}) + \overrightarrow{AD} + (\overrightarrow{AE} + \overrightarrow{ED})$
$= \overrightarrow{AD} + \overrightarrow{AD} + \overrightarrow{AD} = 3\overrightarrow{AD}$



Hence, option (d) is correct.

7. **Solution:** Let $\widehat{AF}$ be a unit vector making angle θ, $\widehat{AF} = AF_x\hat{x} + AF_y\hat{y} + AF_z\hat{z}$
$\widehat{AD} = \hat{y}, \qquad \widehat{AB} = \hat{x}$
$\widehat{AF}.\widehat{AD} = 1.1\cos 120 = -\sin 30 = -0.5 = AF_y$
$\widehat{AF}.\widehat{AB} = 1.1\cos 120 = -\sin 30 = -0.5 = AF_x$
$AF_z = \frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{2}} = \sin\theta = 45^0(in\ degrees)$
Hence, option (d) is correct.

8. **Solution:** $\langle f(\theta)\rangle = \frac{\int f(\theta)d\tau}{\int d\tau}$
$\langle \text{Cos}^2(\theta)\rangle = \frac{\iiint \cos^2(\theta)r^2\sin(\theta)drd\theta d\phi}{\int d\tau}$
$= \frac{\frac{4\pi}{3}R^3}{\frac{4\pi}{3}R^3}\int_0^{\pi/2} \text{Cos}^2(\theta)\sin(\theta)d\theta = \frac{1}{2}\frac{\sqrt{\pi}}{2\times 3\sqrt{\pi}}2.2 = 1/3$
$\langle \text{Cos}^2(\theta)\rangle = \frac{2\pi}{3}2\int_0^{\pi/2} \cos^2(\theta)\sin(\theta)d\theta =$
$\frac{2\pi}{3}2\frac{1}{2}\frac{\sqrt{\pi}}{2\times 3\sqrt{\pi}}2.2 = \frac{4\pi}{9}R^3 = \frac{4\pi}{3}R^3\frac{1}{3} = 1/3\left[\because \frac{4\pi}{3}R^3 = 1\right]$
$\langle \sin^2(\theta)\rangle = \frac{\frac{4\pi}{3}R^3}{\frac{4\pi}{3}R^3}\int_0^2 \sin^3(\theta)d\theta = \frac{1}{2}\frac{\sqrt{\pi}}{3\sqrt{\pi}}2.2 = 2/3$
Hence, option (d) is correct.

9. **Solution:** $\int_{t_1}^{t_2} \vec{r} \times \vec{a}\, dt = \int_{t_1}^{t_2} \vec{r} \times \frac{dv}{dt}dt$
$$\int_{t_1}^{t_2} \vec{r} \times dv = \vec{r} \times \int_{t_1}^{t_2} dv$$
$\vec{r} \times \vec{v}|_{R_1V_1}^{R_2v_2} = \vec{R}_2 \times \overrightarrow{V_2} - \vec{R}_1 \times \overrightarrow{V_1}$
$\vec{I} = \vec{R}_2 \times \overrightarrow{V_2} - \vec{R}_1 \times \overrightarrow{V_1}$
Hence, option (a) is correct.

10. **Solution:**
$$\nabla \times \frac{(x\hat{\imath} + y\hat{\jmath})}{x^2+y^2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{x^2+y^2} & \frac{y}{x^2+y^2} & 0 \end{vmatrix} = \hat{k}\left[\frac{-2xy}{(x^2+y^2)^2} + \frac{2xy}{(x^2+y^2)^2}\right] = 0$$
Hence, option (c) is correct.

11. **Solution:** $\vec{v} = y\vec{\imath} + 3x\vec{\jmath}$
$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & 3x & 0 \end{vmatrix} = \hat{k}(3-1) = 2\hat{k}$$
$\int \vec{v}.dr = \int \nabla \times v.ds = \int 2\,\hat{k}.\hat{k}ds = 2(\pi R^2) = 2\pi \quad [\because R = 1]$
Hence, option (d) is correct.

MATRICES

[2012]

1. For $N \times N$ matrix consisting of all ones,
(a) all eigenvalues = 1
(b) all eigenvalues = 0
(c) the eigenvalues are 1, 2, ……, N
(d) one eigenvalue = N, the others = 0

[2016]

2. Given a matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, which of the following represent cost $\left(\frac{\pi M}{6}\right)$
(a) $\frac{1}{2}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ (b) $\frac{\sqrt{3}}{4}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$
(c) $\frac{\sqrt{3}}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

[2017]

3. Let $\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 11 \end{pmatrix}$ *and* $m = \begin{pmatrix} 10 & 3i \\ -3i & 2 \end{pmatrix}$. similarity, transformation of M to Λ can be performed by
(a) $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3i \\ 3i & 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{9}}\begin{pmatrix} 1 & -3i \\ 3i & 11 \end{pmatrix}$
(c) $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3i \\ -3i & 11 \end{pmatrix}$ (d) $\frac{1}{\sqrt{9}}\begin{pmatrix} 1 & 3i \\ -3i & 1 \end{pmatrix}$